REVISE EDEXCEL GCSE (9–1)

Mathematics

REVISION WORKBOOK

Higher

For the (9–1) exams

Contents

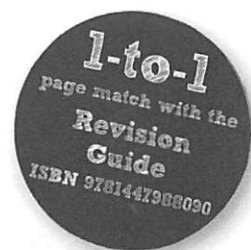1-to-1 page match with the Revision Guide ISBN 9781447988030

Angle properties

Guided

1 Work out the values of x and y.
Give reasons for your answers.

> Angles on a straight line add up to ...°.
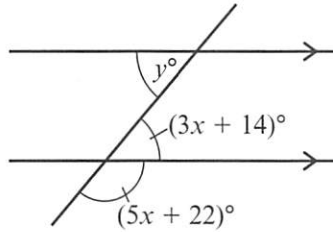
$5x + 22 +$ .... $+$ .... $=$ ....
Because angles on a straight line add up to $180°$
Solve for x

..............$x +$ .... $=$ ....

..............$x =$ ....

$x =$ ....

$y = 3x + 14 =$ .... because

.... angles are equal.   **(4 marks)**

2 Here is an isosceles triangle.
Work out the values of x and y.
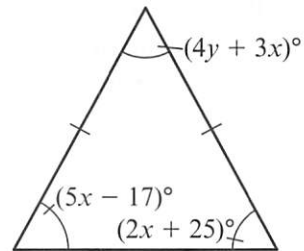
$x =$ ....

$y =$ .... **(5 marks)**

3 The diagram shows a quadrilateral.
Work out the size of the smallest angle.
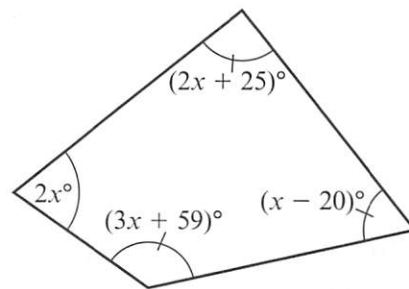
.... ° **(4 marks)**

Solving angle problems

1   Work out the sizes of the angles marked x and y.
    Give reasons for each step of your working.

....
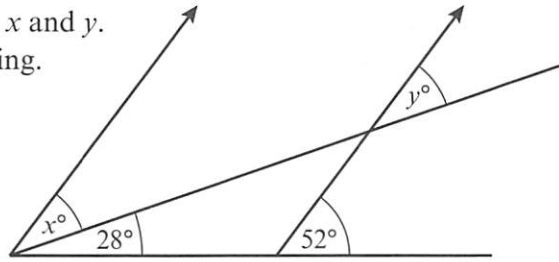
(4 marks)

2   (a)   Work out the size of angle BCD.

.............................°

(3 marks)

    (b)   Show that triangle ADC is isosceles.
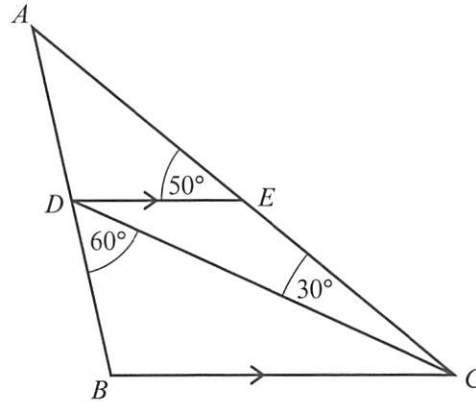
(2 marks)

3   In the diagram, $AB = AC$ and angle ABC
    is bisected by BD.
    Work out the size of the angle marked x.
    Give reasons for each step of your working.
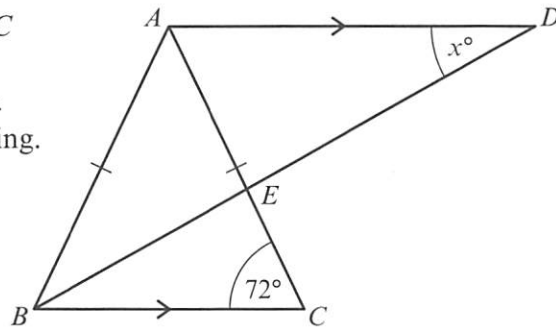
.............................°   **(4 marks)**

4   In the diagram, ABC is a straight line, $AB = BC = BD$ and angle $BAD = x°$.

    (a)   Find the size of angle CBD in terms of x.
       Give your answer in its simplest form.
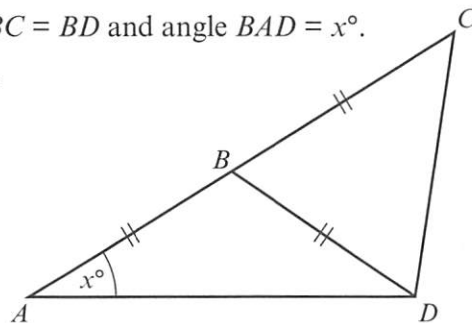
.............................°   **(2 marks)**

    (b)   Work out the size of angle CDA.
       Give reasons for each stage of your working.

.............................°   **(2 marks)**

Angles in polygons

1 The interior angle of a regular polygon is 140°.

(a) Write down the size of the exterior angle of the polygon.

140°
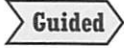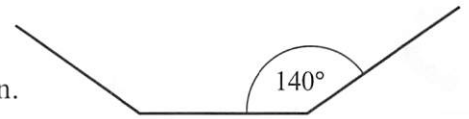
Exterior angle = 180 − ....

             = ...
(1 mark)

(b) Work out the number of sides of the polygon.

Number of sides = 360 ÷ ....

                  = ...
(2 marks)

2 Each diagram shows part of a regular polygon. The size of one interior angle is given. Work out the number of sides for each regular polygon.

(a) 150°

(b) 144°
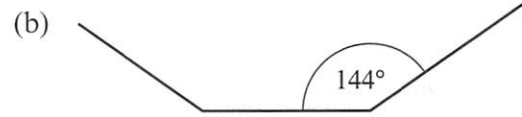
.... **(3 marks)**   .... **(3 marks)**

3 The diagram shows part of a regular octagon.

x
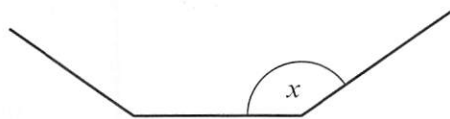
Work out the size of the angle marked x.

$x =$ .... **(3 marks)**

4 The diagram shows part of a regular polygon with n sides.

(a) Work out the value of n.

160°
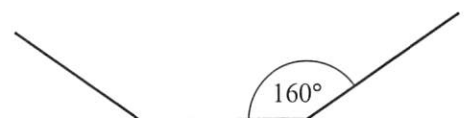
$n =$ .... **(2 marks)**

(b) What is the sum of the interior angles of the polygon?

..........................° **(2 marks)**

5 The diagram shows three sides of a regular hexagon. Show that $x = 30°$.

x

(3 marks)

Pythagoras' theorem

> Guided

1   Work out the lengths of the sides marked with letters in the following triangles.
    Give your answers correct to 3 significant figures.

$$\text{short}^2 + \text{short}^2 = \text{long}^2$$

(a)

7.9 cm x 7.2 cm

(b)

y 10.7 cm   8.3 cm

(c)

5.6 cm   16.7 cm z

$x^2 = \ldots\ldots^2 + \ldots\ldots^2$

$x^2 = \ldots\ldots$

$x = \sqrt{\ldots\ldots}$

$x = \ldots\ldots$ cm **(2 marks)**

$\ldots\ldots^2 = y^2 + \ldots\ldots^2$

$y^2 = \ldots\ldots^2 - \ldots\ldots^2$

$y = \sqrt{\ldots\ldots}$
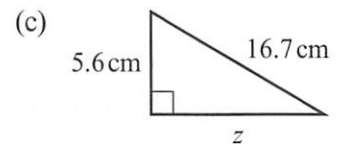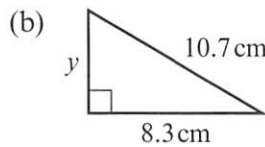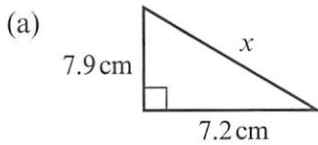
$y = \ldots\ldots$ cm **(2 marks)**

$\ldots\ldots$ **(2 marks)**

2   One end of a rope is tied to the top of a vertical flagpole of height 12.8 m. When the rope is pulled tight, the other end is on the ground 4.2 m from the base of the flagpole. Work out the length of the rope. Give your answer correct to the nearest cm.

Sketch a diagram to help you see what is going on.

$\ldots\ldots$ cm **(2 marks)**

3   Cindy has a rectangular suitcase of length 95 cm and width 72 cm. She wants to put her walking stick into her suitcase. The length of the walking stick is 125 cm. She thinks that the walking stick will fit into one side of her suitcase. Is she correct? Give a reason for your answer.

(2 marks)

4   The diagram shows a small pool with a radius of 2.8 m and a height of 1.5 m.

←2.8 m→   1.5 m
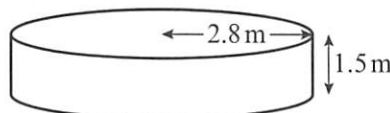
A straight pole is 6 m long. The pole cannot be broken.
Can the pole be totally immersed in the pool?
Give a reason for your answer.

(2 marks)

Trigonometry 1

Guided

1 Work out the size of each of the angles marked with letters. Give each answer to 3 significant figures.

SOH CAH TOA

Start by labelling the sides of the triangle. Then write down the trig ratio that uses these two sides.

(a) opp 8.9 cm hyp x 6.4 cm adj

(b) 9.7 cm y 18.2 cm

(c) 6.5 cm 12.6 cm z

$\tan x = \dfrac{opp}{adj} = \dfrac{............}{............}$

$x = \tan^{-1}............$

$x = ...°$ **(3 marks)**

$............ y = \dfrac{adj}{hyp} = \dfrac{............}{............}$

$y = ...^{-1}............$
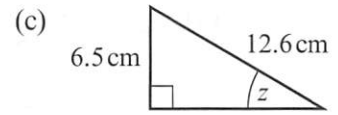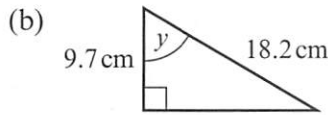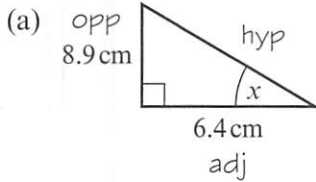
$y = ...°$ **(3 marks)**

.... **(3 marks)**

2 One end of a rope is tied to the top of a vertical mast of height 7.2 m. When the rope is pulled tight, the other end is on the ground 3.7 m from the base of the mast. Work out the angle between the ground and the rope.

........................°
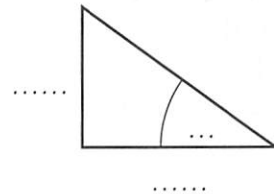
Draw a diagram. **(2 marks)**

3 The diagram shows two right-angled triangles. Work out the size of the angle marked x. Give your answer correct to 3 significant figures.

You will have to use Pythagoras' theorem on the bottom triangle first.

x 25.3 m 7.9 m 18.6 m
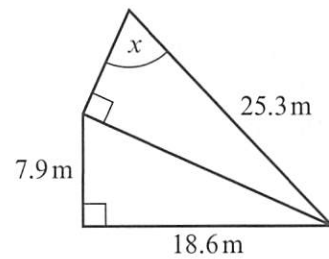
$x = ...$ **(2 marks)**

4 The diagram shows a pitched roof. Hayley wants to use smooth tiles to cover the roof. The smooth tiles can only be used when the angle, x, is at least 17°. Can she use the smooth tiles on her roof? Give a reason for your answer.

3.2 m x 3 m
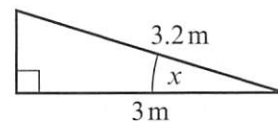
(3 marks)

5 Simon is making a metal slide at home. The diagram shows his slide. He estimates that the slide has to be at an angle of at least 35° for him to go down the slide. Can he go down the slide? Give a reason for your answer.

1.8 m y 2.9 m

(3 marks)

Had a go ☐   Nearly there ☐   Nailed it! ☐
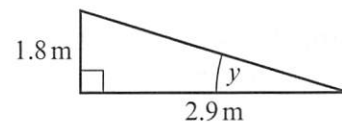
Trigonometry 2

1 Work out the length, in cm, of each of the marked sides.
Give each answer correct to 3 significant figures.

SOH CAH TOA

> **Guided**

Start by labelling the sides of the triangle. Then write down the trig ratio that uses the given and unknown side.

(a)
opp hyp
x
50°
12 cm
adj

(b)
y 18.3 cm
63°

(c)
16.9 cm 28° z

$\tan \ldots\ldots^\circ = \dfrac{opp}{adj} = \dfrac{x}{\ldots\ldots}$

$\sin 63^\circ = \dfrac{opp}{hyp} = \dfrac{y}{18.3}$

$x = \ldots\ldots \times \tan 50^\circ$

$y = \ldots\ldots \times \ldots\ldots 63^\circ$
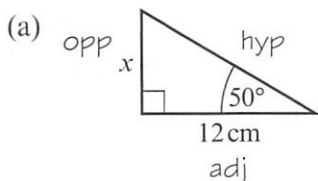
$x = \ldots\ldots$ cm   **(3 marks)**

$y = \ldots\ldots$ cm   **(3 marks)**
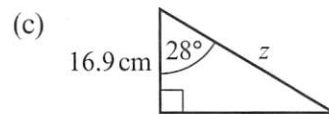
$\ldots\ldots\ldots\ldots$ **(3 marks)**

2 A ladder is 6 m long. The ladder rests against a vertical wall with the foot of the ladder resting on horizontal ground. The ladder makes an angle of 63° with the ground when it is leaning against the wall.
How far does the ladder reach up the wall?

$\ldots\ldots$
$\ldots$
$\ldots\ldots$
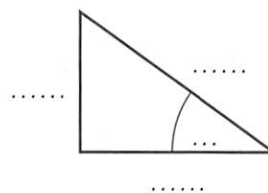
Draw a diagram.

$\ldots\ldots\ldots\ldots$ m **(2 marks)**

3 A tower 40 m high stands at a point A. At a point B on the ground which is level with the foot of the tower, the angle of elevation of the top of the tower is 36°. Work out the distance of B from A.

$\ldots\ldots\ldots\ldots$ m **(2 marks)**

4 The diagram shows a vertical pole standing on horizontal ground. The points A, B and C are in a straight line on the ground. The point D is at the top of the pole so that DC is vertical. The angle of elevation of D from A is 35°.

D
35°
C 12 m B A
$\longleftarrow$ 30 m $\longrightarrow$
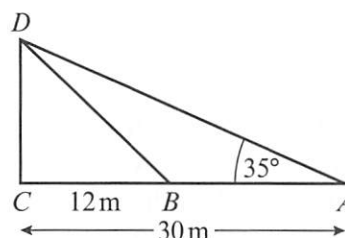
(a) Work out the height of the pole.
Give your answer correct to 3 significant figures.

$\ldots\ldots\ldots\ldots$ m **(2 marks)**

(b) Work out the size of the angle of elevation of D from B.
Give your answer correct to 3 significant figures.

$\ldots\ldots\ldots\ldots$° **(2 marks)**

Solving trigonometry problems

1 Complete the table.

> You must remember these for the exam.

	0°	30°	45°	60°	90°
sin		$\frac{1}{2}$			
cos			$\frac{1}{\sqrt{2}}$		
tan				$\sqrt{3}$	

(5 marks)

Guided

2 Work out the exact length, in cm, of each of the marked sides.

> SOH CAH TOA

> Start by labelling the sides of the triangle. Then write down the trig ratio that uses the given and unknown side.

(a)
hyp
18 cm
opp
60°
x
adj

$$\cos 60° = \frac{adj}{hyp} = \frac{\text{..........}}{\text{..........}}$$

$x = \text{..........} \times \cos 60°$

$x = \text{..........}$ cm   **(3 marks)**
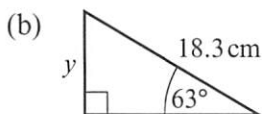
(b)
9.8 cm
y
30°

$$\text{..........} 30° = \frac{opp}{hyp} = \frac{\text{..........}}{\text{..........}}$$

$y = \text{..........} \times \text{..........} 30°$

$y = \text{..........}$ cm   **(3 marks)**

(c)
16.9 cm   45°
z
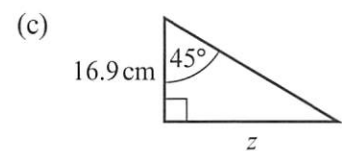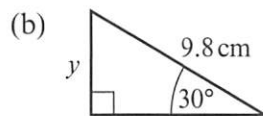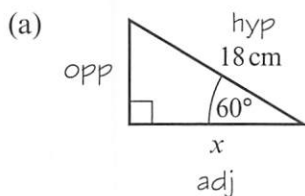
.... **(3 marks)**

Guided

3 Work out the exact size of each of the angles marked with letters.

> SOH CAH TOA

(a)
opp
1 cm
hyp
x
√3 cm
adj

$$\tan x = \frac{opp}{adj} = \frac{\text{..........}}{\text{..........}}$$

$x = \tan^{-1} \text{..........}$

$x = \text{..........}°$ **(3 marks)**

(b)
√3 cm   y   2 cm

$$\text{..........} y = \frac{\text{..........}}{\text{..........}} = \frac{\text{..........}}{\text{..........}}$$

$y = \text{..........}^{-1} \text{..........}$

$y = \text{..........}°$ **(3 marks)**

(c)
10 cm   20 cm
z

.... **(3 marks)**

Guided
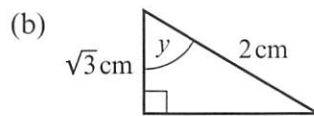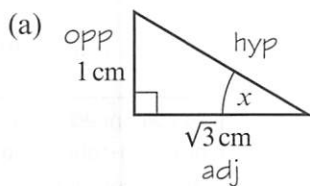
4 Alan is sitting on the ground and his distance from the base of a tower is 30 feet. The angle of elevation from Alan to the top of the tower is 60°. Work out the height of the tower. Leave your answer as an exact value.

......
°
...
......
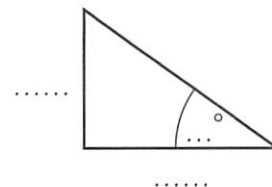
> Draw a diagram.

.... feet   **(3 marks)**

5 A sailor is going to cross a plank of wood from the ground to a boat that is 3 m above the ground level. The length of the plank is 6 m. Show that the angle of elevation is 30°.

(3 marks)

Perimeter and area

Guided

1 The diagram shows the length in centimetres of each side of the rectangle.

(a) Work out the perimeter, P cm, of the rectangle.

$5x + 1$

$2x + 1$ ⬚ $2x + 1$

$29 - 3x$
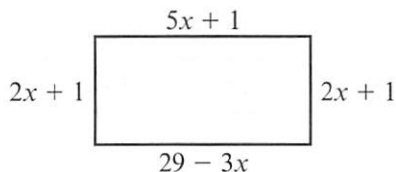
Opposite sides are equal.

$5x + 1 = \ldots\ldots\ldots - \ldots\ldots\ldots x$

$5x + \ldots\ldots\ldots x = \ldots\ldots\ldots - \ldots\ldots\ldots$

$\ldots\ldots\ldots x = \ldots\ldots\ldots$

$x = \ldots\ldots\ldots$ cm

$5x + 1 = 5\ldots\ldots\ldots + 1 = \ldots\ldots\ldots$

$2x + 1 = 2\ldots\ldots\ldots + 1 = \ldots\ldots\ldots$

Perimeter = $\ldots\ldots\ldots + \ldots\ldots\ldots + \ldots\ldots\ldots + \ldots\ldots\ldots = \ldots\ldots\ldots$ cm   **(4 marks)**

(b) Work out the area of the rectangle.

Area = $\ldots\ldots\ldots \times \ldots\ldots\ldots = \ldots\ldots\ldots$ cm^2 **(1 mark)**

2 Here are two shapes with the same area.
Work out the perimeter of the rectangle.

$(x + 2)$ cm

x cm

x cm

24 cm
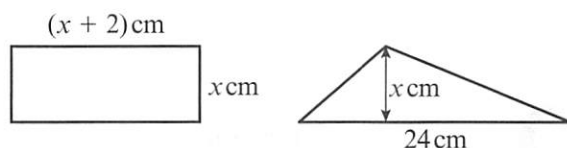
$\ldots\ldots\ldots\ldots\ldots\ldots$ cm   **(4 marks)**

3 The diagram shows a six-sided shape.
All the corners are right angles.
All measurements are given in centimetres.
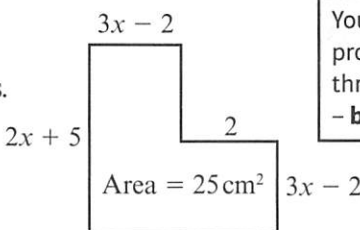
PROBLEM SOLVED!

(a) Show that $6x^2 + 17x - 39 = 0$

$3x - 2$

$2x + 5$

2

Area = 25 cm^2 | $3x - 2$

You will need to use problem-solving skills throughout your exam – **be prepared!**

(3 marks)

(b) Solve the equation $6x^2 + 17x - 39 = 0$

(c) Work out the perimeter of the shape.

$\ldots\ldots\ldots\ldots\ldots\ldots$ **(3 marks)**

$\ldots\ldots\ldots\ldots\ldots\ldots$ cm   **(2 marks)**

4 The length of a rectangle is twice the width of the rectangle.
The length of a diagonal of the rectangle is 25 cm.
Work out the area of the rectangle.
Give your answer as an integer.

x ⬚

$2x$
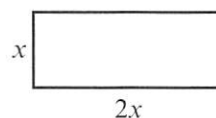
$\ldots\ldots\ldots\ldots\ldots\ldots$ cm^2 **(3 marks)**

Units of area and volume

1 Convert

Guided

(a) $6\,m^2$ into cm^2　(b) $15\,cm^2$ into mm^2　(c) $4\,km^2$ into m^2

$6\,m^2 = 6 \times \ldots\ldots \times \ldots\ldots$

$= \ldots\ldots\ cm^2$ **(2 marks)**　$\ldots\ldots mm^2$ **(2 marks)**　$\ldots\ldots m^2$ **(2 marks)**

(d) $500\,000\,cm^2$ into m^2　(e) $60\,000\,mm^2$ into cm^2　(f) $800\,000\,m^2$ into km^2

$\ldots\ldots m^2$ **(2 marks)**　$\ldots\ldots cm^2$ **(2 marks)**　$\ldots\ldots km^2$ **(2 marks)**

2 Convert

Guided

(a) $22\,m^3$ into cm^3　(b) $28\,cm^3$ into mm^3　(c) $3\,km^3$ into m^3

$22\,m^3 = 22 \times \ldots\ldots \times \ldots\ldots \times \ldots\ldots$

$= \ldots\ldots cm^3$ **(2 marks)**　$\ldots\ldots mm^3$ **(2 marks)**　$\ldots\ldots m^3$ **(2 marks)**

(d) $200\,000\,000\,cm^3$ into m^3　(e) $50\,000\,000\,mm^3$ into cm^3　(f) $420\,000\,000\,m^3$ into km^3

$\ldots\ldots m^3$ **(2 marks)**　$\ldots\ldots cm^3$ **(2 marks)**　$\ldots\ldots km^3$ **(2 marks)**

3 Convert

(a) $200\,000\,cm^3$ into litres　(b) $8\,m^3$ into litres　(c) $12\,m^3$ into litres

$\ldots\ldots$ litres **(2 marks)**　$\ldots\ldots$ litres **(2 marks)**　$\ldots\ldots$ litres **(2 marks)**

4 Work out how many litres of water each tank in the shape of a cuboid can hold.

(a)
20 cm　50 cm　30 cm

(b)
2 m　8 m　6 m
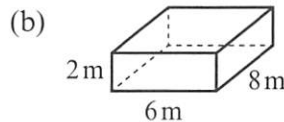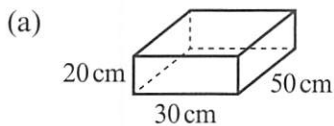
$\ldots\ldots$ litres **(2 marks)**　$\ldots\ldots$ litres **(2 marks)**

5 The pressure in a boiler is $18\,000\,N/m^2$. The area of one end of the boiler is $4000\,cm^2$.
Work out the force on the end of the boiler.

PROBLEM
SOLVED!

You will need to use problem-solving skills throughout your exam – **be prepared!**

$\ldots\ldots$ N **(3 marks)**

Prisms

1 Find the volume of each of these prisms.

Guided

(a)

4 cm
8 cm
3 cm
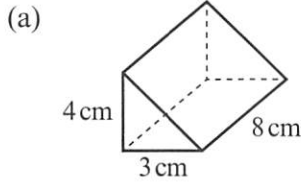
> Volume of a prism = area of cross-section × length.
> You need to learn this formula for your exam.

Volume = ($\frac{1}{2}$ × .... × ....) × ....

Volume = ... cm³   **(2 marks)**

(b)

4 mm   5 mm
6 mm   12 mm

(c)

4 mm
6 mm
12 mm   8 mm
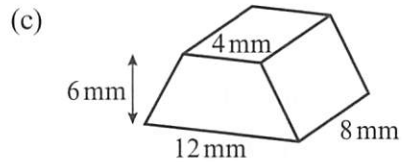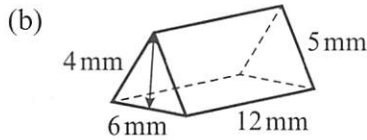
.... mm³ **(2 marks)**   .... mm³ **(2 marks)**

2 Find the surface area of the prisms in Question 1.

(a)

Guided

Surface area =

2($\frac{1}{2}$ × .... × ....) + (.......... × ....) + (.......... × ....) + (.......... × ....)

Surface area = ... cm²   **(3 marks)**

(b)   (c)

.... **(2 marks)**   .... **(3 marks)**

3 The diagram shows a prism.
   The area of the cross-section of the prism is 22 cm².
   The length of the prism is 16 cm.
   Work out the volume of the prism.

22 cm²
16 cm
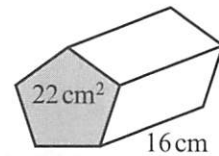
.... cm³ **(2 marks)**

4 The diagram shows a triangular prism and a cuboid.
   They have the same volume.
   Work out the length of x.

5 cm
8 cm   14 cm

7 cm
x cm   4 cm
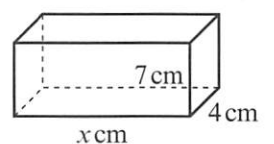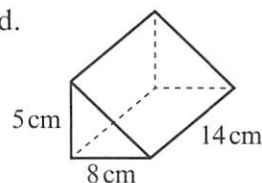
x = ... cm **(4 marks)**

Circles and cylinders

Guided

1   A reel of thread has a radius of 2.5 cm. The thread is wrapped round the reel 200 times. Estimate the length of the thread.
Give your answer correct to 3 significant figures.

$C = 2 \times \pi \times r$

$C = 2 \times \pi \times$ ....

$C =$ .... cm

Length of the thread = 200 × .... = ... cm   **(3 marks)**

2   Bob is rolling a cricket pitch. The length of the pitch is 20.12 m.
The roller has a radius of 8 cm.
Show that the roller makes 40 complete turns.

(3 marks)

3   The diagrams show two identical squares.
Shape **A** is a quarter of a circle shaded inside the square.
Shape **B** is two identical semicircles shaded inside the square.
Show that the area of the region shaded in **A** is equal to area of the region shaded in **B**.
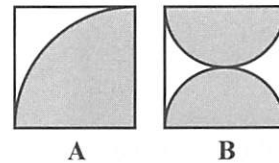
(3 marks)

4   Object **A** is a cylinder with radius 15 cm and height 18 cm.
Object **B** is a cube with side length 24 cm.
Show that the volume of the cube is greater than the volume of the cylinder.
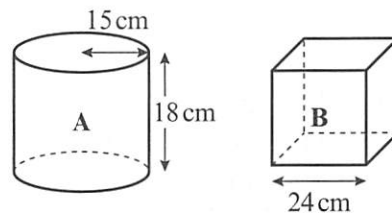
(4 marks)

5   Object A is a cylindrical vase with radius 9 cm and height 21 cm.
Object B is a vase with a square base of length 8 cm and height 42 cm.
Which vase has the greatest surface area? You must show your working.
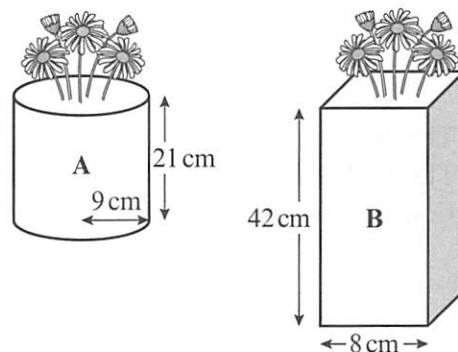
(4 marks)

Sectors of circles

1 Work out the arc lengths of these sectors of circles. Give your answers correct to 3 significant figures.

> You are only calculating the curved length in this question.

Guided

(a)

80° 4 cm
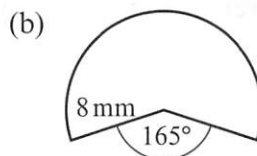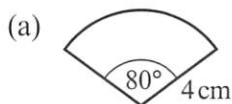
Arc length = $2 \times \pi \times r \times \dfrac{\ldots\ldots}{360}$

$\qquad = 2 \times \pi \times \ldots\ldots \times \dfrac{\ldots\ldots}{360}$

$\qquad = \ldots\ldots$ cm **(3 marks)**

(b)

8 mm 165°

.... mm **(3 marks)**

2 Work out the perimeters of these sectors of circles. Give your answers correct to 3 significant figures.

> Find the arc length then add the two radii.

(a)

85° 7 cm

.... cm **(4 marks)**

(b)

9 mm 155°
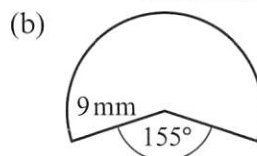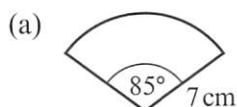
.... mm **(4 marks)**

3 Work out the areas of the sectors in Question 2. Give your answers correct to 3 significant figures.

Guided

(a) $A = \pi \times r \times r \times \dfrac{\ldots\ldots}{360}$

$\qquad = \pi \times \ldots\ldots \times \ldots\ldots \times \dfrac{\ldots\ldots}{360}$

$\qquad = \ldots\ldots$ cm² **(3 marks)**

(b)

.... mm **(3 marks)**

4 The diagram shows a sector of circle with radius 10.4 cm. Calculate the area of the shaded segment *ABC*. Give your answer correct to 3 significant figures.

> Work out the area of the sector.

> Work out the area of the triangle.

B
A C
10.4 cm .... 10.4 cm
120°
O
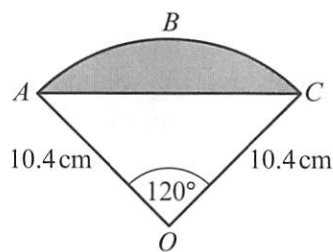
.... cm² **(4 marks)**

5 The diagram shows a sector of a circle, centre *O*, radius 10 cm. The arc length of the sector is 15 cm. Calculate the area of the sector.

10 cm .... 15 cm
O .... 10 cm
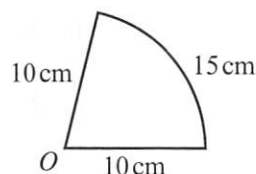
.... cm² **(4 marks)**

Volumes of 3D shapes

Guided

1 Work out the volumes of these solid shapes.
Give your answers correct to 3 significant figures.

(a)

15 cm
4 cm
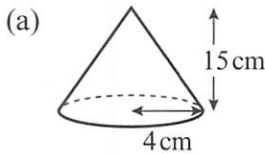
$V = \frac{1}{3} \times \pi \times r^2 \times h$

$= \frac{1}{3} \times \pi \times \text{............}^2 \times \text{............}$

$= \text{............ } cm^3$ **(2 marks)**

(b)

12 cm

.... cm³ **(2 marks)**

(c)

17 cm
←5 cm→ 5 cm
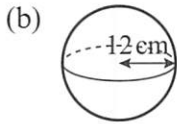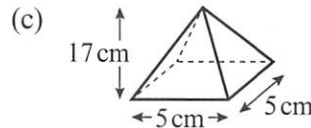
.... cm³ **(2 marks)**

2 Work out the volumes of these solid shapes. Give your answers correct to 3 significant figures.

(a)

4 cm
10 cm

.... cm³ **(3 marks)**

(b)

6 cm
10 cm
5 cm
5 cm
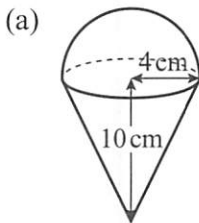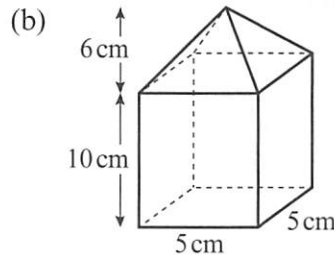
.... cm³ **(3 marks)**

PROBLEM SOLVED!

3 A cylinder has base radius x cm
and height $3x$ cm.
A cone has base radius x cm
and height h cm.
The volume of the cylinder and
the volume of the cone are equal.
Find h in terms of x.
Give your answer in its simplest form.

3x cm
x cm

h cm
x cm
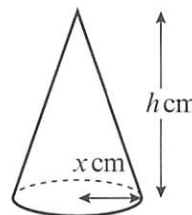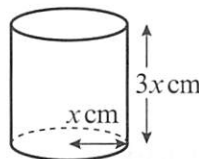
You will need to use
problem-solving skills
throughout your exam
– **be prepared!**

$h = $ .... **(4 marks)**

4 The diagram shows a cylinder and a sphere.
The radius of the base of the cylinder is x cm
and the height of the cylinder is h cm.
The radius of the sphere is x cm.
The volume of the cylinder is three times the volume of the sphere.
Show that the height, h, is 4 times the radius, x.

h
x
←x→
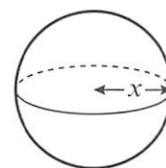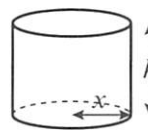
(4 marks)

Surface area

Guided

1 Work out the total surface areas of the following shapes.
Give your answers correct to 3 significant figures.

(a)

Surface area $= (\pi \times r^2) + (\pi \times r \times l)$

$= (\pi \times \ldots\ldots^2) + (\pi \times \ldots\ldots \times \ldots\ldots)$

$= \ldots\ldots \text{ cm}^2$ **(2 marks)**

16 cm

4 cm

(b)

12 cm

(c)

7 cm
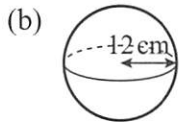
.... cm² **(2 marks)**

.... cm² **(2 marks)**

Guided

2 Work out the total surface areas of the following shapes.
Give your answers correct to 3 significant figures.

(a)

Surface area $= cone + hemisphere$

$= (\pi \times r \times l) + \frac{1}{2}(4 \times \pi \times r^2)$

$= (\pi \times \ldots\ldots \times \ldots\ldots) + \frac{1}{2}(4 \times \pi \times \ldots\ldots^2)$

$= \ldots\ldots \text{ cm}^2$ **(3 marks)**

4 cm

10 cm

(b)

3 m
3 m
30 m
3 m

(c)

21 cm
5 cm
3 cm
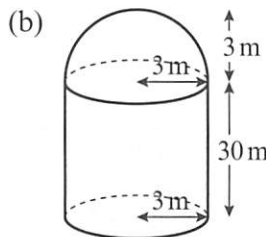
.... cm² **(3 marks)**
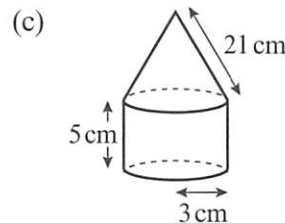
.... cm² **(3 marks)**

PROBLEM SOLVED!

3 The diagram shows a cone with vertical height 15 cm and base diameter 16 cm. Work out the curved surface area of the cone.

You will need to use Pythagoras' theorem to find the slant height. Sketch the right-angled triangle you need to use.

15 cm

16 cm
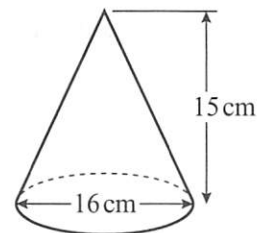
You will need to use problem-solving skills throughout your exam – **be prepared!**

.... cm² **(4 marks)**

Plans and elevations

Guided

1 The diagrams show the different elevations of a shape.
Using this information sketch the 3D shape.

(a)

plan   side   front

(2 marks)

(b)

plan   side   front
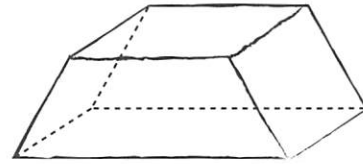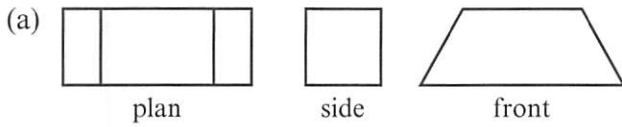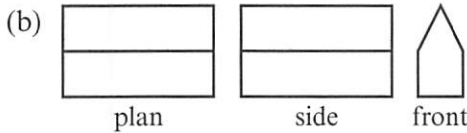
(2 marks)

PROBLEM SOLVED!

2 The diagram shows a triangular prism.
Accurately construct a plan and elevations
for this prism.

You will need to use
problem-solving skills
throughout your exam
– **be prepared!**

4 cm

4.5 cm

3 cm

side

front
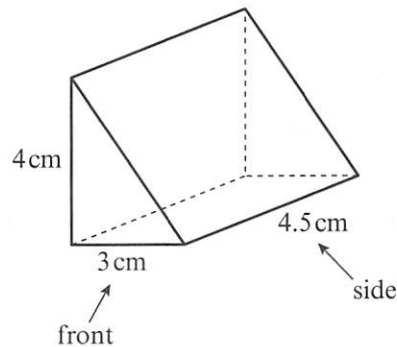
(4 marks)

3 Here is a 3D diagram of the first two patterns in a
series of cube-shaped bricks.
Sketch a side elevation of the arrangement of
cubes of the next pattern in the series.

← side elevation
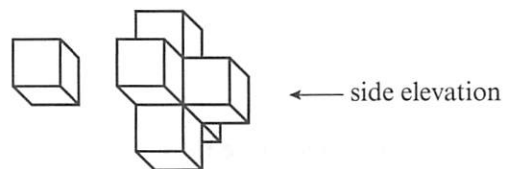
(2 marks)

Translations, reflections and rotations

1
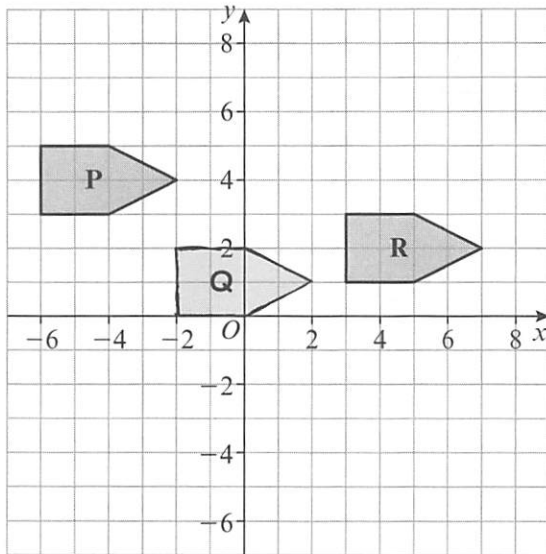
(a) Translate shape **P** by the vector $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$
Label the new shape **Q**.

$\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ means 4 units to the right and 3 units down.

(2 marks)

(b) Reflect shape **R** in the line $y = x$
Label the new shape **S**.

Draw the line $y = x$

(2 marks)

(c) Rotate shape **P** by 180° about the point $(-1, 0)$.
Label the new shape **T**.

(2 marks)

2
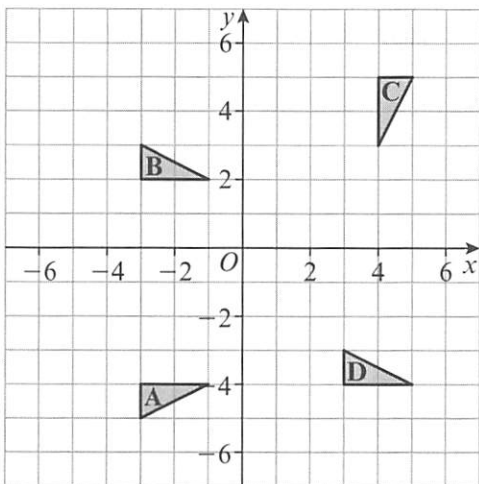
(a) Describe fully the single transformation that will map shape **A** onto shape **B**.

.... **(2 marks)**

(b) Describe fully the single transformation that will map shape **B** onto shape **C**.

.... **(2 marks)**

(c) Describe fully the single transformation that will map shape **B** onto shape **D**.

.... **(2 marks)**

Enlargement

1 (a) Shape **B** is an enlargement of shape **A**.
      Find the scale factor of the enlargement.

Guided

.... ÷ .... = ...

(1 mark)

3 cm

2 cm ☐ A

15 cm

☐ B ☐ 10 cm
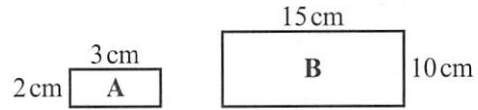
(b) Enlarge the triangle below by scale factor 2.
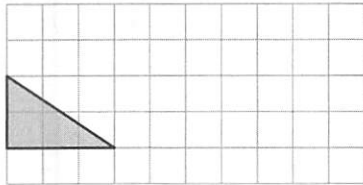
> No centre of enlargement is given so the enlarged shape can be placed anywhere on the grid.

(1 mark)

2 Enlarge triangle **A** by scale factor 3, centre (0, 0).
    Label the new shape **B**.

Guided
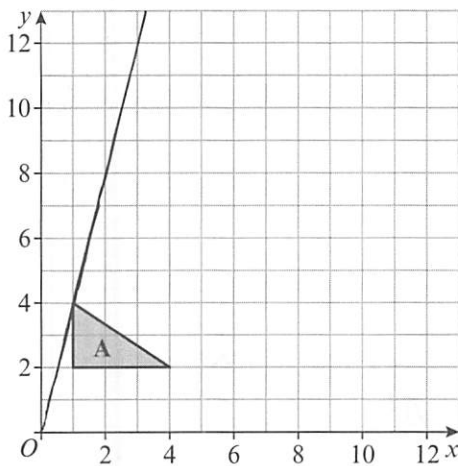
> Draw lines from the centre of enlargement through each vertex of the triangle.

(2 marks)

3 Triangle **A** is shown on the grid.
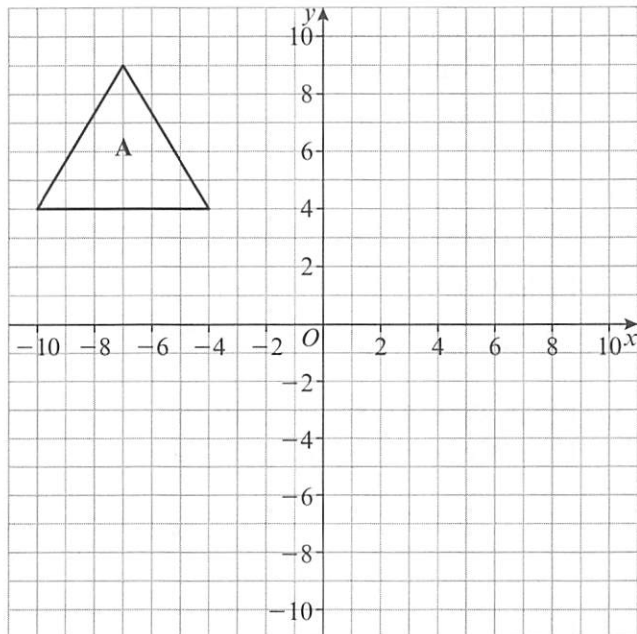
(a) Enlarge triangle **A** by scale factor $\frac{1}{2}$ with centre of enlargement (1, −3).
    Label the image **B**.   **(2 marks)**

(b) Enlarge triangle **A** by scale factor $-\frac{1}{2}$ with centre of enlargement (−3, −2).
    Label the image **C**.   **(3 marks)**

Combining transformations

1 (a) Reflect shape **A** in the line $y = 1$
Label the new shape **B**.

> **Guided**

Draw the line $y = 1$ **(2 marks)**

(b) Rotate shape **B** by 180° about the point (5, 1).
Label the new shape **C**.

Use tracing paper to rotate the shape.   **(2 marks)**
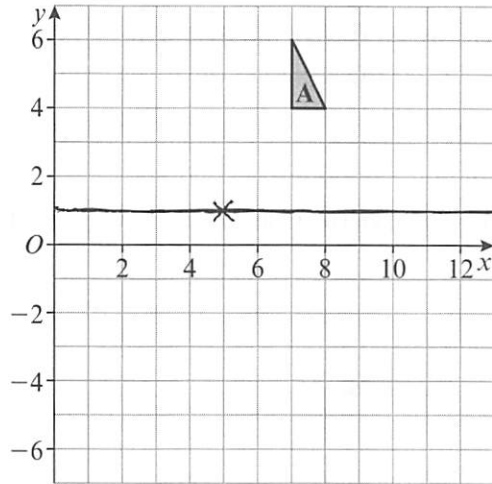
(c) Describe fully the single transformation which maps shape **A** onto shape **C**.

.... **(2 marks)**

2 (a) Translate shape **P** by the vector $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$
Label the new shape **Q**. **(2 marks)**

(b) Rotate shape **Q** by 180° about the point (7, 7).
Label the new shape **R**. **(2 marks)**

(c) Describe fully this single transformation which maps shape **P** onto shape **R**.
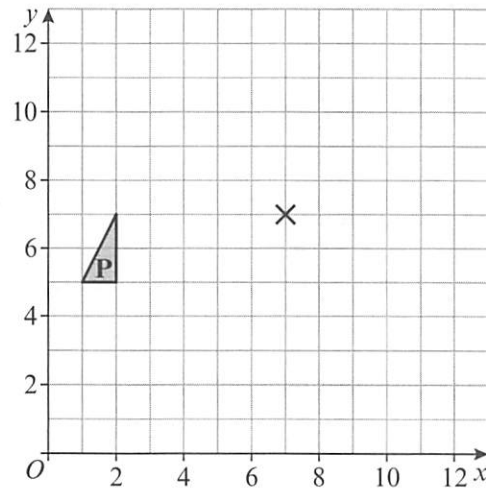
.... **(2 marks)**

3 (a) Reflect triangle **T** in the line $y = x$
Label the new shape **U**. **(2 marks)**

(b) Rotate shape **U** 180° about (0, 0).
Label the new shape **V**. **(2 marks)**

(c) Describe fully the single transformation which maps shape **T** onto shape **V**.
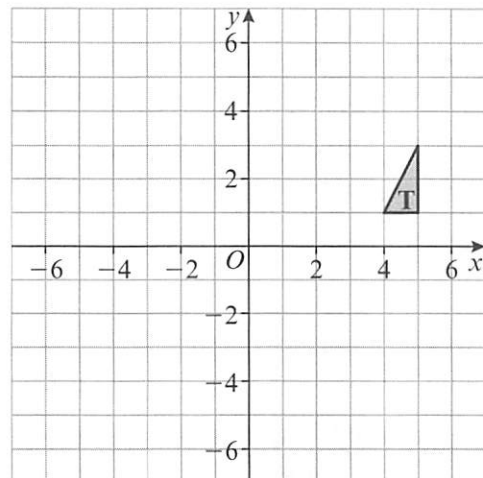
.... **(3 marks)**

Bearings

1 The diagram shows the position of each of three telephone masts, A, B and C.
The bearing of mast B from mast A is 034°. Mast C is due east of mast B.
The distance from mast A to mast B is equal to the distance from mast B to mast C.
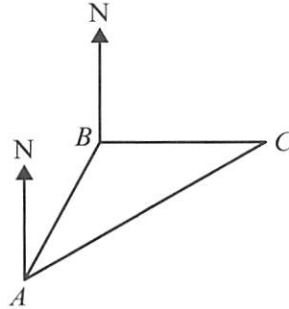
Work out the bearing of mast C from mast A.

.... **(3 marks)**

2 A submarine S is 3.5 km due west of a naval base B.
A frigate F is 2.4 km due north of the submarine S.
Find the bearing of the frigate F from the naval base B.
Give your answer correct to 3 significant figures.

Work out angle SBF.

$$\tan x = \frac{opp}{adj}$$

$$\tan x = \frac{\text{............}}{\text{............}}$$

$$x = \tan^{-1}\frac{\text{............}}{\text{............}}$$

$$x = \text{............}°$$
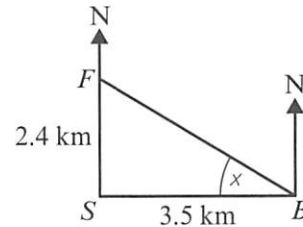
The bearing of the frigate F from the naval base B = ...° + ...° = ...°

(4 marks)

3 A, B and C are three villages.
A is 10.9 km due west of B.
B is 8.5 km due south of C.
Calculate the bearing of A from C.
Give your answer correct to 3 significant figures.
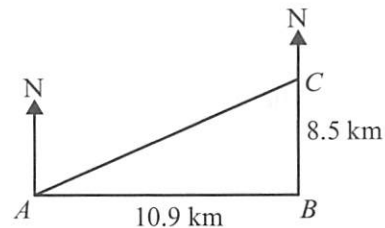
..........................° **(4 marks)**

Scale drawings and maps

1 Arthur uses a scale of 1 : 300 to make a model of an aeroplane.

> **Guided**

(a) The wingspan of the model is 5 cm.
Work out the wingspan of the
aeroplane.

300 × 5 = ...cm

= ....m   **(2 marks)**

(b) The length of the aeroplane is 45 m.
Work out the length of the model.

> The length of the model must be smaller
> than the length of the real aeroplane.

.... cm **(2 marks)**

2 Here is a scale drawing of Andrew's garden patio.
He wants to cover the patio using slabs.
Each slab is 50 cm by 50 cm square.

Andrew buys 120 slabs. Does he buy enough slabs
to completely cover his patio?
You must show all your working.
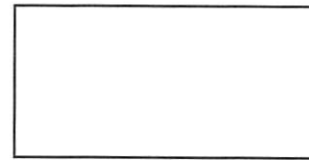
1 cm represents 2 m

(3 marks)

3 The map below shows a small island drawn using a scale of 1 : 500 000.
There are three beacons at *A*, *B* and *C*.

John drives from *A* to *B*, from *B* to *C* and from *C* back to *A*.
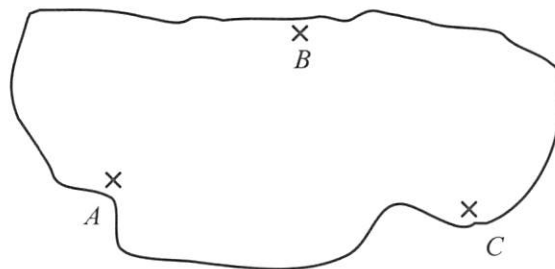
(a) What is the total distance he drives?

.... km **(3 marks)**

(b) The bearing of *D* from *A* is 60°.
The bearing of *D* from *C* is 290°

> Add point *D* on the map

Work out the distance of *D* from *B*. Give your answer in km.

.... km **(3 marks)**

Constructions 1

Guided

1 Use a ruler and compasses to construct the perpendicular bisector of AB.

A ———————— B
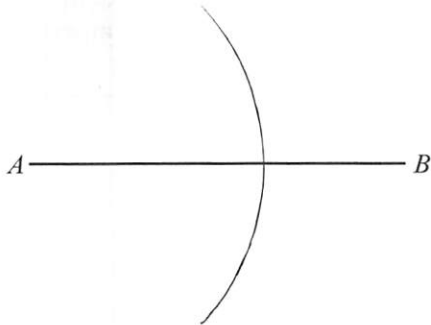
> Do not rub out the arcs you make when using your compass.

> 1. Draw an arc, centre A, with radius more than half the length of AB above and below the line segment AB.
> 2. Draw another arc, centre B, with the same radius, above and below the line segment AB.
> 3. Draw a line through the two points where the arcs cross each other above and below the line segment AB.

(2 marks)

Guided

2 Use a ruler and compasses to construct the perpendicular to the line segment AB that passes through the point T.

A —— ✕ —— B
          T
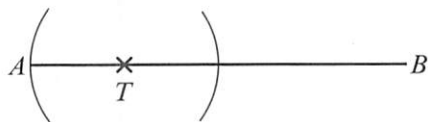
> You must show all your construction lines.

> First draw two arcs, centre T, with the same radius to cross AB on either side of T.

(2 marks)

3 Use a ruler and compasses to construct the perpendiculars from P to the line segments AB and CD. You must show all your construction lines.

✕ P

D

A

C

B

(3 marks)

93
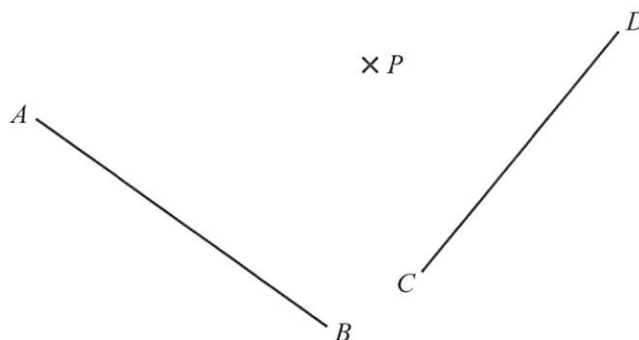
Constructions 2

1 Use a ruler and compasses to construct a triangle with sides of lengths 3.5 cm, 4 cm and 5 cm.

> Do not rub out the arcs you draw when using your compasses.

Guided

> 1. Draw a horizontal line of 5 cm and label it *AB*.
> 2. Set the compasses at 3.5 cm, then draw an arc with centre *A*.
> 3. Set the compasses at 4 cm, then draw an arc with centre *B*.
> 4. Draw lines from the point of intersection to *A* and *B*.

$$A \text{———} 5\,cm \text{———} B$$
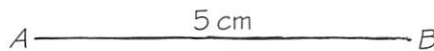
(2 marks)

2 Use a ruler and compasses to construct the bisector of angle *ABC*.
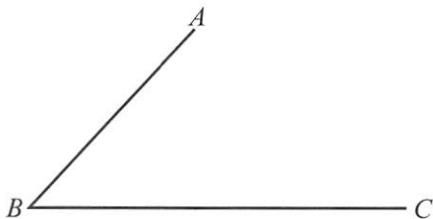
> 1. Draw an arc, centre *B*, to cross *AB* at *P* and *BC* at *Q*.
> 2. Draw two arcs, centres *P* and *Q*, with the same radius. The two arcs cross.
> 3. Draw a line through where the arcs cross to *B*.

A

$B \text{———} C$

(2 marks)

3 Use a ruler and compasses to construct a 60° angle at *A*. You must show all your construction lines.

$$A \text{———} B$$

(2 marks)

4 Use a ruler and compasses to construct a 45° angle at *A*. You must show all your construction lines.

> Start by constructing a perpendicular to *AB* through *A*.

$$A \text{———} B$$

(2 marks)

Loci

1 Draw the locus of all points that are exactly 2 cm from the line AB.

> 1. Draw a circle of radius 2 cm with centre A.
> 2. Draw a circle of radius 2 cm with centre B.
> 3. Draw two parallel lines 2 cm above and below the line AB.

A ————————— B

(2 marks)

2 The diagram shows the boundary of a rectangular garden, $ABCD$. A dog is tied to the corner C with a rope of length 6 m. Shade the region where the dog can reach.

A D

B C
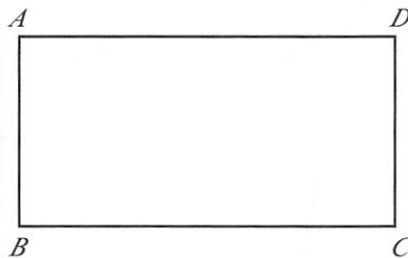
> 1. Use the scale to set the compasses at the required distance.
> 2. Draw an arc with centre C.
> 3. Shade the required region.

1 cm represents 2 m

(2 marks)

3 P, Q and R represent three radio masts on a map. Signals from mast P can be received 100 km away, and from masts Q and R 75 km away. Shade the region in which signals can be received from all three masts.

1 cm represents 25 km

× Q

P ×

× R

(3 marks)

4 ABC is a triangle. Shade the region inside the triangle which is both less than 3 cm from B and closer to the line AC than the line AB.

C

A

B

(3 marks)

95
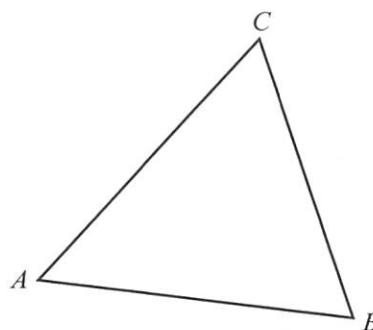
Congruent triangles

1 Show that triangle *ABD* is congruent to triangle *CDB*.

Guided

BD is common to both triangles.

Angle ABD = angle BDC because they are

.... angles.

AB = DC since opposite sides of a

.... are equal.

Hence, triangles ABD and CDB are congruent.

The condition that has been satisfied is ....
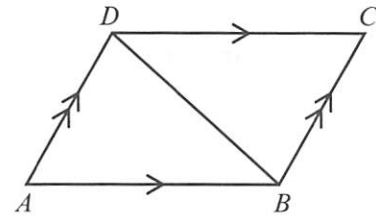
(3 marks)

2 *ABC* is an isosceles triangle, in which *AB = AC*.
L and *M* are points on *AB* and *AC* such that *AL = AM*. Prove that triangles *BLC* and *CMB* are congruent.
State the reason for congruency.

> You will need to use problem-solving skills throughout your exam – **be prepared!**

PROBLEM SOLVED!
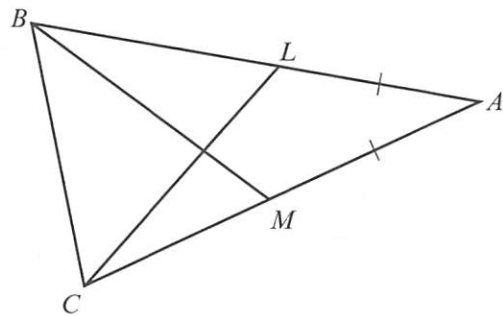
(3 marks)

3 *ABC* is an equilateral triangle. *D* lies on *BC*.
AD is perpendicular to *BC*.

(a) Prove that triangle *ADC* is congruent to triangle *ADB*.
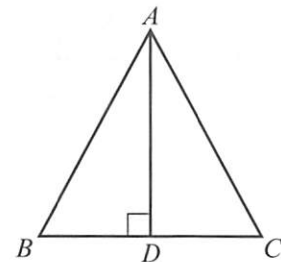
(3 marks)

(b) Hence, prove that $DC = \frac{1}{2}AC$.

(2 marks)

4 In the diagram, the lines *AC* and *BD* intersect at *E*. *AB* and *DC* are parallel. Prove that triangles *ABE* and *CDE* are congruent.
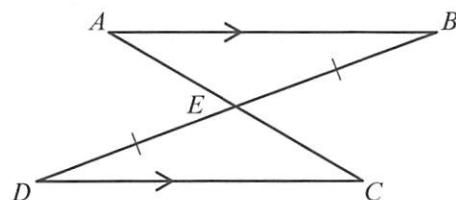
(4 marks)

Similar shapes 1

Guided

1 The two triangles ABC and PQR are mathematically similar.

(a) Work out the length of PR.
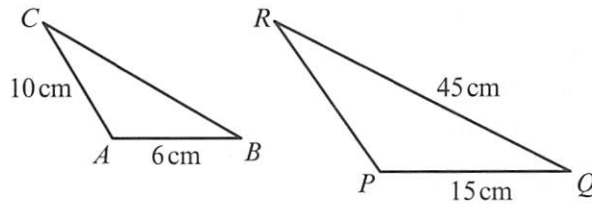
$$\frac{PR}{10} = \frac{15}{\ldots\ldots}$$

$$PR = \frac{15}{\ldots\ldots} \times \ldots\ldots\ldots = \ldots\ldots\ldots \text{ cm}$$

(2 marks)

> Use the fact that corresponding sides are in the same ratio.

(b) Work out the length of BC.

.... cm **(2 marks)**

2 Triangle ABC is mathematically similar to triangle ADE. Work out the length of DE.

> Separate into two triangles.
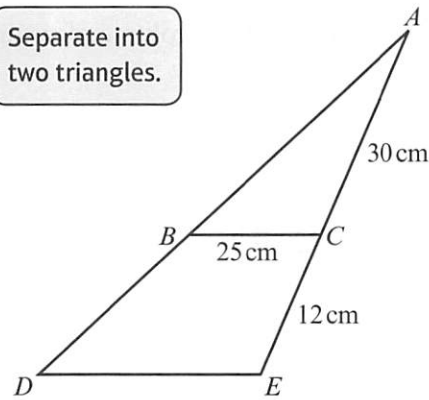
.... cm **(3 marks)**

3 Triangles ABC and ACD are mathematically similar. $AB = 8$ cm and $AC = 10$ cm. Calculate the length of AD.
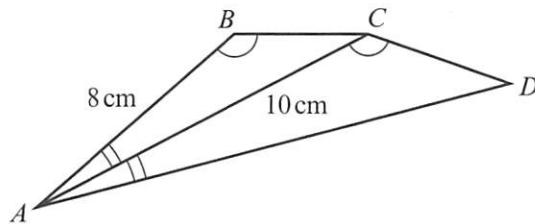
.... cm **(3 marks)**

4 AB is parallel to DC. The lines AC and BD intersect at E. $AB = 6$ cm, $AE = 8$ cm, $DE = 14$ cm, $EC = 20$ cm. Work out the length of BE.
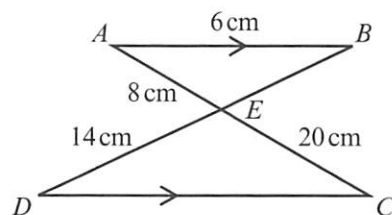
.... cm **(3 marks)**

Similar shapes 2

Guided

PROBLEM SOLVED!

1 Two perfume bottles are mathematically similar in shape. The smaller bottle is 4 cm high and holds 40 millilitres of perfume. The larger bottle is 6 cm high. What volume of perfume, in millilitres, will the larger bottle hold?
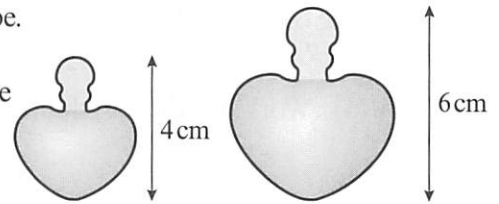
> You will need to use problem-solving skills throughout your exam – **be prepared!**

> Work out the linear scale factor of the heights.

4 cm

6 cm

Linear scale factor = 6 ÷ .... = ...

Volume scale factor = ... × .... × .... = ...

Volume of large bottle = 40 × .... = ... millilitres   **(3 marks)**

2 The diagram shows two similar wedges, **A** and **B**.

A   5 cm   B   15 cm
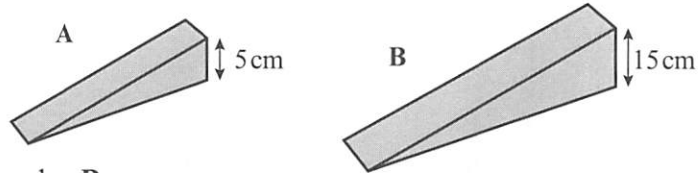
(a) Wedge **A** has a volume of 90 cm³.
     Work out the volume of wedge **B**.

.... cm³ **(2 marks)**

(b) Wedge **B** has a total surface area of 162 cm². Work out the surface area of wedge **A**.

.... cm² **(2 marks)**

3 **A** and **B** are two mathematically similar solid shapes. The total surface area of shape **X** is 464 cm² and the total surface area of shape **Y** is 725 cm².
   The volume of shape **X** is 2752 cm³.
   Work out the volume of shape **Y**.

.... cm³ **(3 marks)**

4 Two cones, **L** and **M**, are mathematically similar. The total surface area of cone **L** is 24 cm² and the total surface area of cone **M** is 384 cm². The height of cone **L** is 45 cm.

(a) Work out the height of cone **M**.

.... cm **(2 marks)**

(b) The volume of cone **M** is 17 152 cm³. Work out the volume of cone **L**.

.... cm³ **(2 marks)**

The sine rule

1 Find the lengths of the sides marked with a letter.

(a)

10.4 cm

23°

56°

x
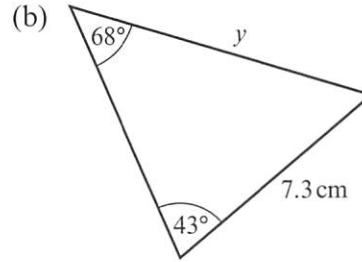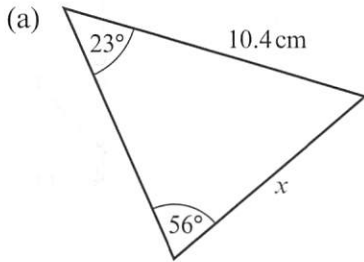
Using the sine rule

$$\frac{x}{\sin 23°} = \frac{10.4}{\sin 56°}$$

$$x = \frac{10.4}{\sin 56°} \times \sin 23°$$

x = ... cm   **(3 marks)**

(b)

68°

y

7.3 cm

43°

y = ... cm   **(3 marks)**

2 Find the sizes of the angles marked with a letter.

(a)

x

25 cm

16.2 cm

49°
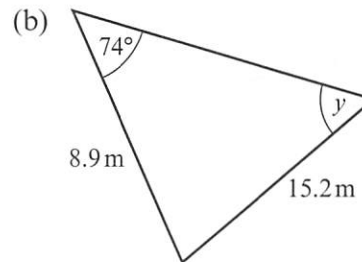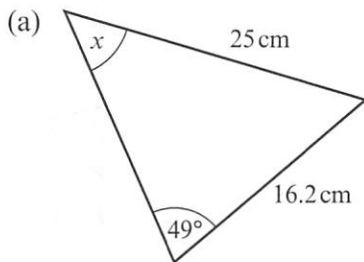
Using the sine rule

$$\frac{\sin x}{16.2°} = \frac{\sin 49°}{25}$$

$$\sin x = \frac{\sin 49°}{25} \times 16.2$$

x = ...°   **(3 marks)**

(b)

74°

y

8.9 m

15.2 m

y = ...°   **(3 marks)**

3 *ABC* is a triangle.
Calculate the length of *AB*.
Give your answer correct to 3 significant figures.

C   22.4 m   B

65°

58°

A
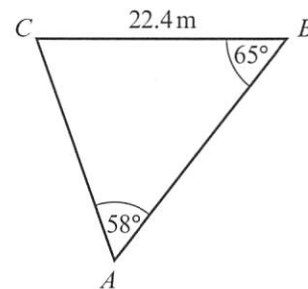
.... m   **(3 marks)**

4 The diagram shows a flagpole held by 2 ropes. From point *B*, the angle of elevation to the top of the flagpole *F* is 60°. From point *A*, the angle of elevation to the top of the flagpole *F* is 72°. The distance *AB* is 6.7 m. Find the height of the flagpole.

F

O   A   B
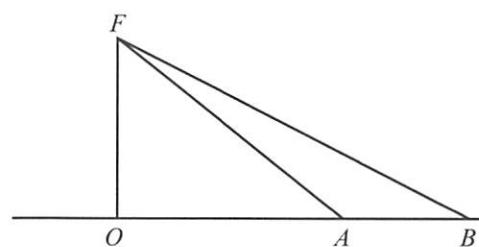
....   **(5 marks)**

The cosine rule

1 Find the lengths of the sides marked with a letter.

> Guided

(a)

23° 22.3 cm
16.5 cm
x
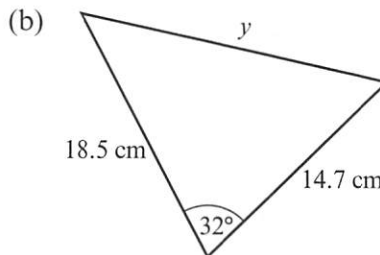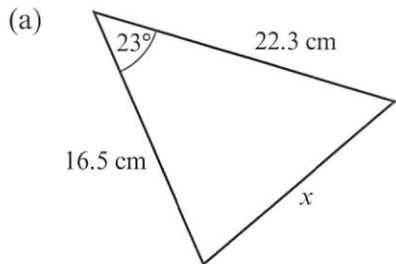
Using the cosine rule

$x^2 = 16.5^2 + 22.3^2 - (2 \times \text{..................}$

$\times \text{..................} \times \cos \text{..................})$

$x^2 = \text{..................}$

$x = \text{..................} cm$ **(3 marks)**

(b)

y
18.5 cm   14.7 cm
32°

$y = \text{..........................} cm$ **(3 marks)**

2 Find the sizes of the angles marked with a letter.

> Guided

(a)

x 23.2 cm
15.6 cm
18.3 cm
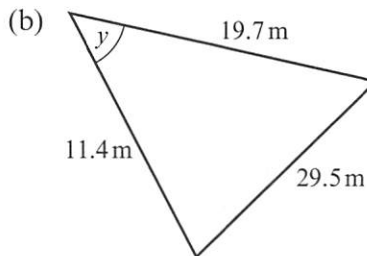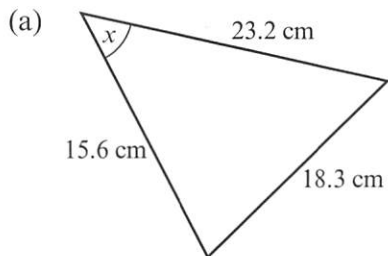
Using the cosine rule

$$\cos x = \frac{\text{..........}^2 + \text{..........}^2 - \text{..........}^2}{2 \times \text{..........} \times \text{..........}}$$

$x = \cos^{-1} \text{..................}$

$x = \text{..................}°$ **(3 marks)**

(b)

y 19.7 m
11.4 m
29.5 m

$y = \text{..........................}°$ **(3 marks)**

3 In triangle ABC, $AC = 9$ cm, $BC = 17$ cm and angle $ACB = 68°$. Calculate the length of AB. Give your answer correct to 3 significant figures.

A 9 cm C
68°
17 cm
B
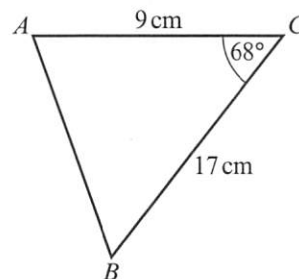
$\text{..........................}$ cm   **(3 marks)**

4 Two boats leave a port P. Boat A sails on a bearing of 074° for 40 km and stops. Boat B sails on a bearing of 145° for 60 km and stops. How far apart are the two boats when they have both stopped?

> Draw a diagram with angles and distances correctly labelled.

$\text{..........................}$ km   **(4 marks)**

Triangles and segments

1 Work out the areas of the triangles.

(a)

23°   22.3 cm

16.5 cm
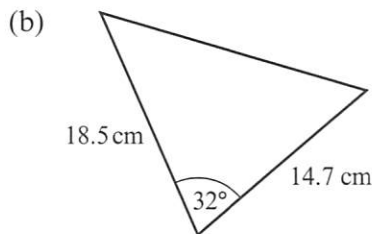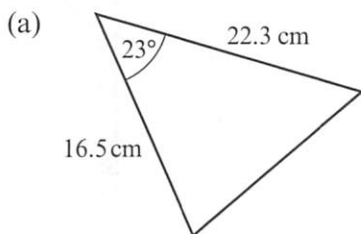
Using the formula for the area

Area $= \dfrac{1}{2}ab \sin C$

$= \dfrac{1}{2} \times \,......... \times .... \times \sin ....$

$= ....\,\text{cm}^2$ **(2 marks)**

(b)

18.5 cm

14.7 cm

32°

.... cm²   **(2 marks)**

2 Work out the areas of the shaded regions.

(a)

15 cm

40°
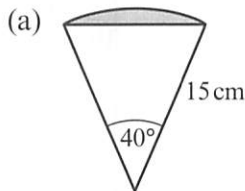
Using the formula for the area of a triangle

Area $= \dfrac{1}{2}ab \sin C$

$= \dfrac{1}{2} \times \,......... \times .... \times \sin ....$

$= ....\,\text{cm}^2$

Using the formula for the area of a sector

Area $= \pi r^2 \times \dfrac{\theta}{360}$

$= \pi \times ....^2 \times \dfrac{.........}{360} = ...\,\text{cm}^2$

Shaded area = Area of sector − Area of triangle

$= .... - .... = \text{cm}^2$ **(5 marks)**

(b)

105°   23 m
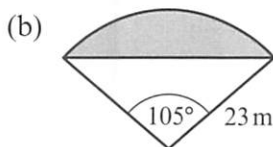
.... m²   **(5 marks)**

3 The diagram shows an equilateral triangle *ABC* with sides of length 10 m. *D* is the midpoint of *AB*. *E* is the midpoint of *AC*. *ADE* is a sector of a circle with centre *A*.

B

D   10 m

A   *E*   *C*

(a) Work out the area of the triangle *ABC*.
Give your answer correct to 3 significant figures.
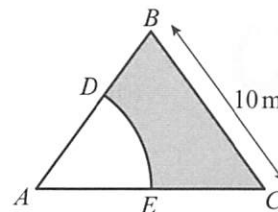
.... m²   **(2 marks)**

(b) Hence, or otherwise, work out the area of the shaded region.
Give your answer correct to 3 significant figures.

.... m²   **(3 marks)**

Pythagoras in 3D

1 A cuboid has length 3 cm, width 4 cm and height 12 cm.
Work out the length of PQ.

> **Guided**

Work out the diagonal
of the bottom surface.
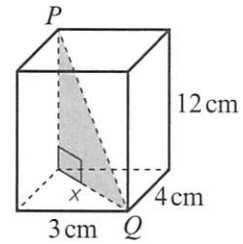
$x^2 = $ ....^{2} + ...^{2}

$x = \sqrt{\text{.................}}$

$x = $ ....cm

$PQ^2 = $ ....^{2} + ...^{2}

$PQ = $ ....cm

(3 marks)

2 The diagram represents a cuboid $ABCDEFGH$.
$AB = 8$ cm, $BC = 10$ cm, $AE = 3.5$ cm.

Calculate the length of AG. Give your
answer correct to 3 significant figures.
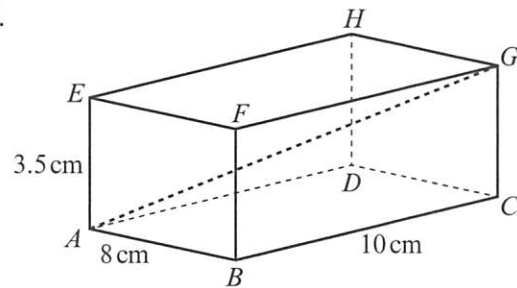
.... cm **(3 marks)**

3 The diagram shows a pyramid. The apex of the
pyramid is V. Each of the sloping sides of the
pyramid is 8 cm. The base of the pyramid is a
regular hexagon with sides of length 3 cm.
O is the centre of the base.

Calculate the height of V above the base of
the pyramid. Give your answer correct to
3 significant figures.
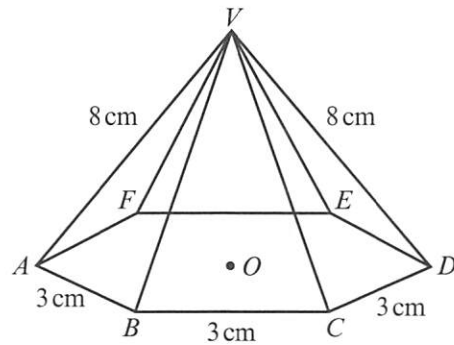
.... cm **(4 marks)**

4 A box has length 30 cm, width 8 cm and height 11 cm.
Show that a stick of length of 35 cm will not fit completely in the box.

(4 marks)

Trigonometry in 3D

1 *ABCDEFGH* is a solid cuboid with sides of 6 cm, 5 cm and 12 cm as shown.

> **Guided**
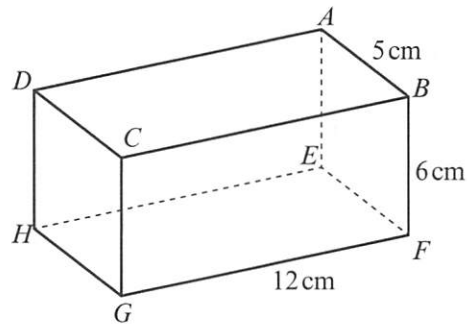
(a) Work out the length of *FH*.

> Work out the diagonal of the bottom surface.

$FH^2 = \ldots\ldots\ldots^2 + \ldots\ldots\ldots^2$

$FH = \sqrt{\ldots\ldots\ldots}$ cm

(2 marks)

(b) Work out the angle *FHB*. Give your answer correct to 3 significant figures.

$\tan x = \dfrac{opp}{adj} = \dfrac{\ldots\ldots\ldots}{\ldots\ldots\ldots}$

$x = \ldots\ldots\ldots$

$= \ldots\ldots\ldots°$

(2 marks)

2 *VABCD* is a right pyramid on a square base. *V* is vertically above the centre of the square.
VA = *VB* = *VC* = *VD* = 40 cm and *AB* = 30 cm.
Work out the angle between the edge *VA* and the base *ABCD*.
Give your answer correct to 3 significant figures.
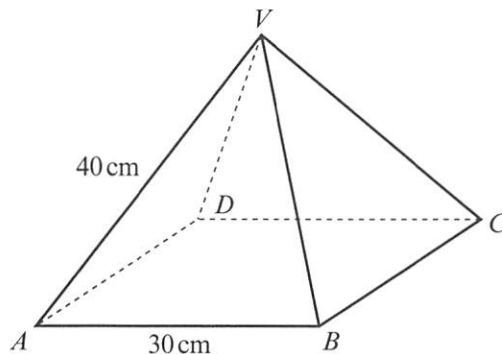
$\ldots\ldots\ldots°$ **(4 marks)**

3 The diagram shows a door wedge with a rectangular horizontal base *PQRS* and a rectangular sloping face *PQTU*.
PQ = 4.2 cm and angle *TQR* = 12°.
The height *TR* is 3 cm.
Calculate the length of the diagonal *PT*.
Give your answer correct to 3 significant figures.
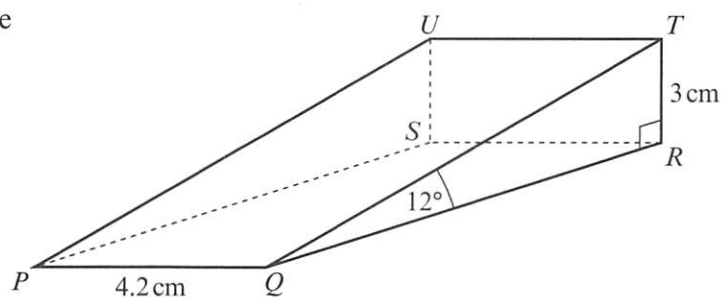
$\ldots\ldots\ldots$ cm **(4 marks)**

Circle facts

Guided

1 In the diagram, A and B are points on the circle. AC and BC are tangents to the circle and meet at C. Work out the angle ACB. Give your answer correct to 3 significant figures.
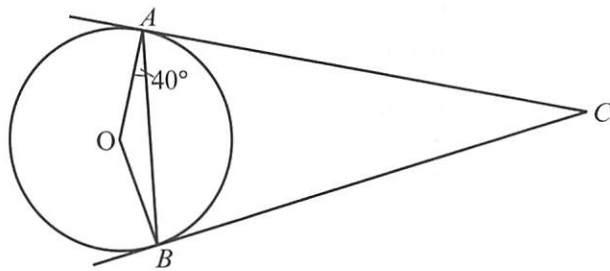
Triangle AOB is an .... triangle.

Angle AOB = 180° − (........................° + ...°)

Angles in the quadrilateral OACB add up to ....°

Angle OAC = Angle OBC = ...°

Angle ACB = ...° − (........................° + ...° + ...°)

Angle ACB = ...°   **(3 marks)**

2 The diagram shows a circle with centre O. A, L, B and M are points on the circumference of the circle. MN and LN are tangents to the circle. AB is parallel to MN and angle LNM is 48°.
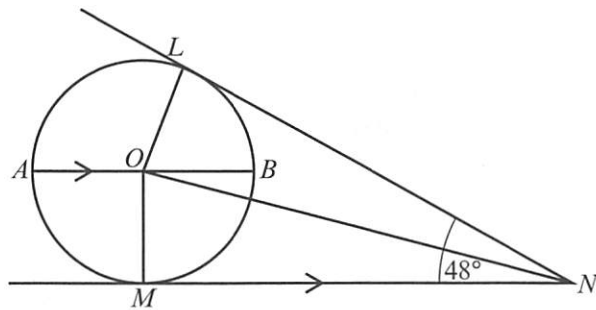
(a) (i) Write down the size of angle OLN.

........................° **(1 mark)**

(ii) Give a reason for your answer.

.... **(1 mark)**

(b) Work out the size of angle LOB.

........................° **(2 marks)**

3 A and B are two points on a circle with centre O. BN is a tangent. Angle ABN is x. Prove that the size of angle AOB is $2x$.
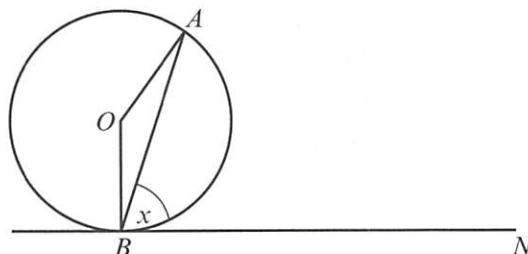
(4 marks)

Circle theorems

1 A, B, C and D are points on a circle.
Find the size of angle ACD. Give reasons for your answer.

> **Guided**
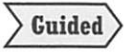
$ACD =$ ....° because angles in the same

.... are ....
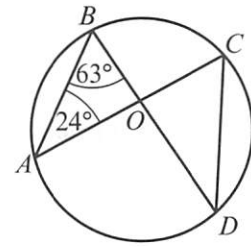
(2 marks)

2 In the diagram, A, B, C and D are points on the circle centre O.

> **Guided**

(a) Work out the size of the angle marked x.
Give a reason for your answer.

$x =$ ....° because the angle at the centre is

.... the angle at the circumference.
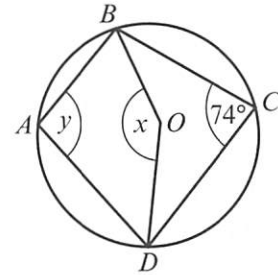
(2 marks)

(b) Work out the size of the angle marked y. Give a reason for your answer.

.... (2 marks)

3 In the diagram, B, D and E are points on the circle centre O. ABC is a tangent to the circle. BE is the diameter of the circle.

(a) Work out the size of angle ABD.
Give a reason for your answer.
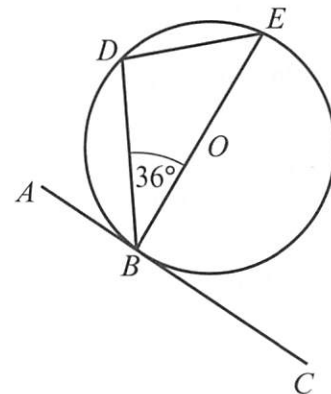
.... (2 marks)

(b) Work out the size of angle DEB. Give a reason for your answer.

.... (2 marks)

4 A, B and C are points on the circumference of the circle centre O.
The line XCY is the tangent at C to the circle. $AB = CB$.
Work out the size of angle OCB.
Give reasons for your answer.
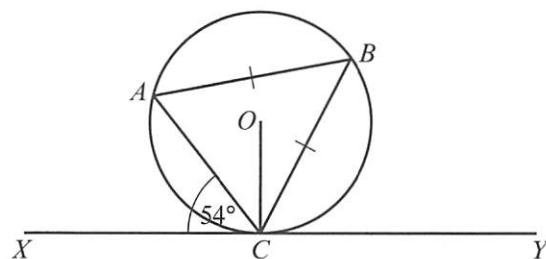
.... (5 marks)

Vectors

1 Write each vector as a column vector.

> **Guided**

(a) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ \ \end{pmatrix}$ (b) $\overrightarrow{BA} = \begin{pmatrix} \ \\ -5 \end{pmatrix}$

    (1 mark)   **(1 mark)**

(c) $\overrightarrow{CD} = \begin{pmatrix} \ \\ \ \end{pmatrix}$ (d) $\overrightarrow{DC} = \begin{pmatrix} \ \\ \ \end{pmatrix}$

    (1 mark)   **(1 mark)**

(e) $\overrightarrow{EF} = \begin{pmatrix} \ \\ \ \end{pmatrix}$ (f) $\overrightarrow{FE} = \begin{pmatrix} \ \\ \ \end{pmatrix}$

    (1 mark)   **(1 mark)**
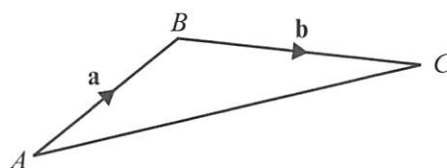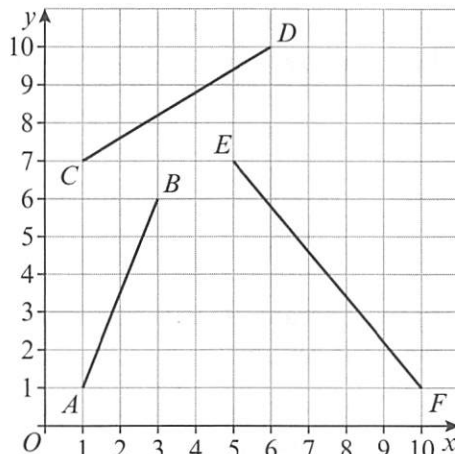
2 The diagram shows triangle ABC.
Write down the following vectors in
terms of **a** and **b**.

> **Guided**

(a) $\overrightarrow{AC} = \mathbf{a} +$ .... **(1 mark)**   (b) $\overrightarrow{CA} =$ .... **(1 mark)**

> Underline any vectors you write down.

3 $ABCD$ is a parallelogram.
AB is parallel to DC.
AD is parallel to BC.
Write down the following vectors in terms of **p** and **q**.

(a) $\overrightarrow{AC}$ (b) $\overrightarrow{CA}$

.... **(1 mark)**   .... **(1 mark)**

(c) $\overrightarrow{DB}$ (d) $\overrightarrow{BD}$
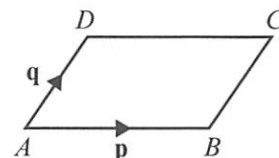
.... **(1 mark)**   .... **(1 mark)**

4 $\overrightarrow{OB} = 3\mathbf{a} + 2\mathbf{b}$

$\overrightarrow{OA} = 2\mathbf{a} + \mathbf{b}$

Write down the following vectors in terms of **a** and **b**.
Give your answers in their simplest form.
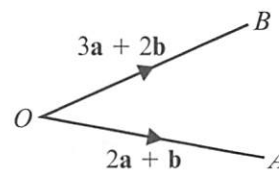
(a) $\overrightarrow{AB}$

.... **(2 marks)**

(b) $\overrightarrow{BA}$

.... **(2 marks)**

Vector proof

1 $OABC$ is a parallelogram. M is the midpoint of AB.
N is the midpoint of BC.
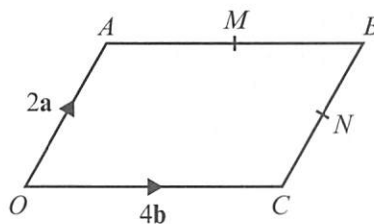
> **Guided**

   (a) Find these vectors in terms of **a** and/or **b**.

     (i) $\overrightarrow{MB}$

     $\overrightarrow{MB}$ is .... of $\overrightarrow{AB}$ so $\overrightarrow{MB}$ = ...   **(1 mark)**

     (ii) $\overrightarrow{MN}$

     $\overrightarrow{MN}$ = ... − ....   **(1 mark)**

   (b) Show that CA is parallel to MN.

                                     (2 marks)

2 $ABCD$ is a triangle. $\overrightarrow{AC}$ = **a** and
$\overrightarrow{AB}$ = **b**. M is the midpoint of AB.
CNB is a straight line where
$CN:NB = 1:4$
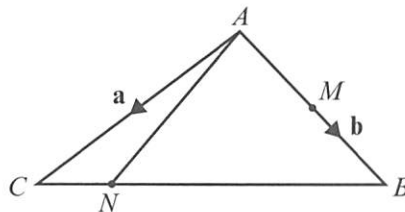
   (a) Find these vectors in terms of **a**
     and/or **b**.

     (i) $\overrightarrow{CB}$ (ii) $\overrightarrow{CN}$ (iii) $\overrightarrow{AN}$

   .... **(1 mark)**   .... **(1 mark)**   .... **(1 mark)**

   (b) Show that $\overrightarrow{MN} = \frac{1}{10}(8\mathbf{a} - 3\mathbf{b})$

                                       (3 marks)

3 $OABC$ is a parallelogram. $\overrightarrow{OA}$ = **a** and $\overrightarrow{OD}$ = **c**.
BCE is a straight line where $BC:CE = 1:2$
D is the midpoint of OC.
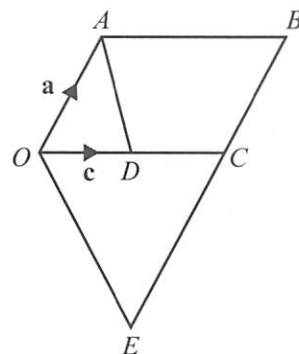
   Work out the ratio of the lengths $AD:OE$

                    .... **(5 marks)**

Problem-solving practice 1

1 The radius of the base of a cone is x cm and its vertical height is h cm.
The radius of a sphere is x cm.
The volume of the cone and the volume of the sphere are equal.
Show that the height, h, is 4 times the radius, x.

(4 marks)

2 A rescue helicopter is stationed at a point X. An emergency call is received and the helicopter flies 16 km on a bearing of 040° to point Y. A second emergency call is received and the helicopter then flies 30 km on a bearing of 115° to arrive at point Z. The helicopter flies back to point X.

(a) Work out the distance that the helicopter has to fly from point Z to point X. Give your answer correct to 3 significant figures.

.... km **(3 marks)**

(b) Work out the bearing on which the helicopter has to fly to from point Z to point X. Give your answer as three-figure bearing.

...........................° **(3 marks)**

3 Two cones, L and M, are mathematically similar. The ratio of the volume of cone L to the volume of cone M is $27 : 125$. The surface area of cone M is 450 cm². Show that the surface area of cone L is 162 cm².

(3 marks)

Problem-solving practice 2

4 The diagram shows a pyramid with a horizontal rectangular base $PQRS$. $PQ = 18$ cm, $QR = 12$ cm and $MT = 16$ cm. M is the midpoint of the line PR. The vertex T is vertically above M.
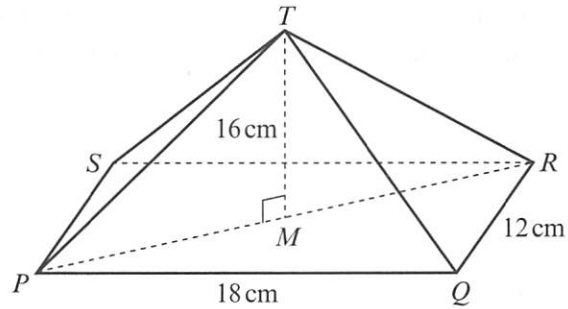
Work out the size of angle between TP and the base $PQRS$.
Give your answer correct to 3 significant figures.

......................° **(4 marks)**

5 $ABCD$ is a trapezium.
AB is parallel to DC.
E is the point on the diagonal such that $DE = \frac{1}{4}DB$.

Prove that BC is parallel to AE.
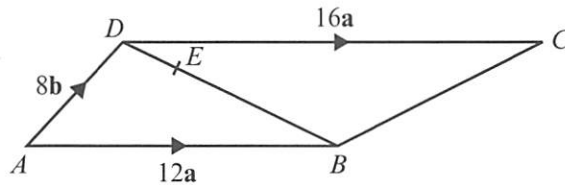
(5 marks)

6 AOD is the diameter of a circle with centre O and radius 12 cm. ABC is an arc of the circle. Calculate the area of the shaded segment.
Give your answer correct to 3 significant figures.
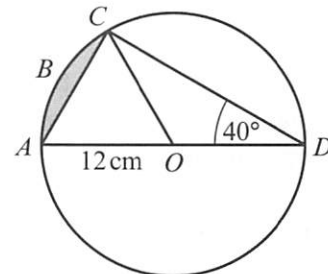
......................cm² **(5 marks)**

65. Speed

1 425 km/h
2 8.9 m/s
3 93.75 km/h
4 240 km
5 Pavan's speed is $35 \div 0.25 = 140$ km/h
6 Jane's speed is 40 km/h and Carol's speed is 29.1 km/h, so Carol has the lower average speed
7 100 m speed is 6.75 m/s and 200 m speed is 6.72 m/s, so 100 m race had the faster speed

66. Density

1 0.875 g/cm³
2 147 g
3 10 375 cm³
4 432 g
5 5666.4 g
6 $950 \div 96 = 9.9$, so Gavin is not correct; it is bronze
7 7.374 g/cm³

67. Other compound measures

1 400 N/m²
2 18 750 N/m²
3 0.006 25 m²
4 Volume = $120\,000 \div 2 = 60\,000$
   Time = $60\,000 \div 250 = 240$ s = 4 min
5 No, population is $12\,170 \times 0.24 = 2921$
6 It takes 13 hours and 20 min; she gets there at 10.20pm; yes, can reach the peak by 11pm

68. Proportion and graphs

1 1170
2 7.5
3 (a) 0.16   (b) 5
   (c) The graph is a straight line through the origin; there is a constant increase / as extension increases force increases
4 (a) 2   (b) 50
   (c) As pressure increases volume decreases

69. Proportionality formulae

1 (a) 1050 s   (b) 600 s
2 14.4 g
3 (a) $T = 22x$ (b) 308 N   (c) 14.5 cm
4 (a) 84   (b) 245

70. Harder relationships

1 (a) $d = 4t^2$ (b) 144   (c) 4
2 (a) $S = \dfrac{3240}{f^2}$ (b) 50.625
3 (a) $I = 0.75\sqrt{P}$ (b) 5.25   (c) 44.4
4 2

71. Problem-solving practice 1

1 Nile: £32.57
   T-bay: £37.08
   Nile is cheaper
2 (a) 8 parts = £240
       1 part = £30
       Angus = $4 \times 30 = £120$
   (b) $(4 \times 30) + (7 \times 30) + (12 \times 30) = £690$
3 30 ml

72. Problem-solving practice 2

4 $325 \div 3\frac{7}{15} = 93.75$ km/h
   Yes, he breaks the speed limit
5 1000 g
6 Yes, he saves £17 674.68
7 Yes, she is correct

GEOMETRY & MEASURES

73. Angle properties

1 $x = 18$, because angles on a straight line add up to 180, and $y = 68$, because alternate angles are equal so angle $DBC = 72 \div 2 = 36$

2 $x = 14$ and $y = 8$
3 17°

74. Solving angle problems

1 $x = 24$ and $y = 24$
2 (a) 20°   (b) angle DAC = angle DCA
3 $x = 36°$
   Base angles in an isosceles triangle are equal
   Angle ADB = angle DBC are alternate angles
4 (a) $2x$
   (b) Angles in a triangle add up to 180°
       Base angles in an isosceles triangle are equal
       Angle $CDB = (180 - 2x) \div 2 = 90 - x$
       and angle $BDA = x$
       Angle $CDA = 90 - x + x = 90$

75. Angles in polygons

1 (a) 40°   (b) 9
2 (a) 12   (b) 10
3 135
4 (a) 18   (b) 2880°
5 Exterior angle = 60°
   Interior angle = 120°
   $x + 120 + x = 180$
   $x = 30$

76. Pythagoras' theorem

1 (a) 10.7   (b) 6.75   (c) 15.7
2 14.37
3 Suitcase diagonal = 119 cm
   No, it will not fit
4 Diagonal of cross-section of pool = 5.8 m
   No, it cannot be totally immersed

77. Trigonometry 1

1 (a) 54.3   (b) 57.8   (c) 31.1
2 62.8°
3 53.0
4 She can use smooth tiles on her roof, because x is 20.4 and hence the angle is greater than 17°
5 $y = 31.8$, so no he cannot

78. Trigonometry 2

1 (a) 14.3   (b) 16.3   (c) 19.1
2 5.35 m
3 55.1 m
4 (a) 21.0 m   (b) 60.3°

79. Solving trigonometry problems

1

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	–

2 (a) 9   (b) 4.9   (c) 16.9
3 (a) 30°   (b) 30°   (c) 30°
4 $30\sqrt{3}$
5 Height above ground is opposite angle of elevation and length of plank is hypotenuse.
   Angle of elevation = $\sin^{-1}(3 \div 6) = \sin^{-1}(0.5) = 30°$

80. Perimeter and area

1 (a) 53 cm   (b) 148 cm²
2 44 cm
3 (a) $2(3x - 2) + (3x - 2)(2x + 5) = 25$
       $6x^2 + 17x - 39 = 0$
   (b) $x = 1.5$ and $-13/3$ (c) 25
4 250 cm²

81. Units of area and volume

1. (a) 60 000   (b) 1500   (c) 4 000 000
   (d) 50   (e) 600   (f) 0.8
2. (a) 22 000 000   (b) 28 000
   (c) 3 000 000 000   (d) 200
   (e) 50 000   (f) 0.42
3. (a) 200   (b) 8000   (c) 12 000
4. (a) 30   (b) 96 000
5. 7200

82. Prisms

1. (a) 48   (b) 144   (c) 384
2. (a) 108   (b) 204   (c) 339.4
3. 352 cm³
4. $x = 10$ cm

83. Circles and cylinders

1. 3140
2. 40.02 = 40
3. Area $A = \frac{1}{4} \times \pi \times x^2 = \frac{1}{4}\pi x^2$
   Area $B = 2 \times \frac{1}{2} \times \pi \times \left(\frac{x}{2}\right)^2 = \frac{1}{4}\pi x^2$
4. Cylinder volume = $\pi \times 15^2 \times 18 = 12\,723$ cm³
   Cube volume = $24 \times 24 \times 24 = 13\,824$ cm³
   Volume of cube is greater
5. Area of A = $(2 \times \pi \times 9 \times 21) + (\pi \times 9^2) = 1442$ cm²
   Area of B = $(8 \times 8) + (4 \times 8 \times 42) = 1408$ cm²
   Vase A has the greatest surface area

84. Sectors of circles

1. (a) 5.59   (b) 27.2
2. (a) 24.4   (b) 50.2
3. (a) 36.3   (b) 145
4. 66.4
5. 75 cm²

85. Volumes of 3D shapes

1. (a) 251   (b) 7240   (c) 142
2. (a) 302   (b) 300
3. $\pi \times x^2 \times 3x = \frac{1}{3} \times \pi \times x^2 \times h$
   So $h = 9x$
4. $\pi \times x^2 \times h = 3 \times 4\frac{1}{3} \times \pi \times x^3$
   So $h = 4x$

86. Surface area

1. (a) 251   (b) 1810   (c) 462
2. (a) 226   (b) 650   (c) 320
3. 628

87. Plans and elevations

1. (b)

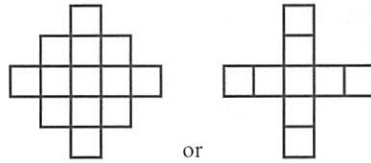2. Plan =   rectangle base = 3 cm and height = 4.5 cm

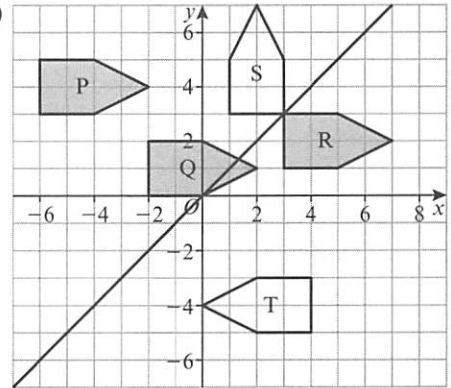   Front =   triangle with base 3 cm and height 4 cm

   Side =   rectangle base = 4.5 cm and height = 4 cm

3.   or

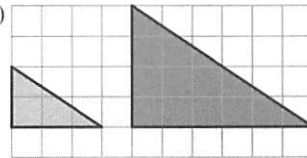## 88. Translations, reflections and rotations

1. (b), (c)

2. (a) Reflection in the line $x = -1$
   (b) Rotation 90° clockwise about (2, 0)
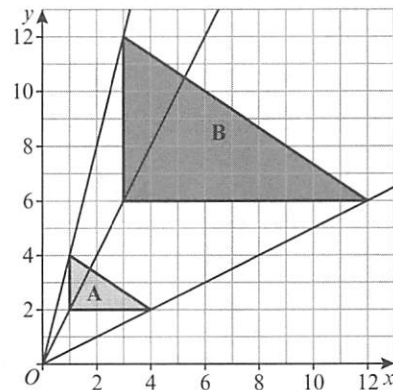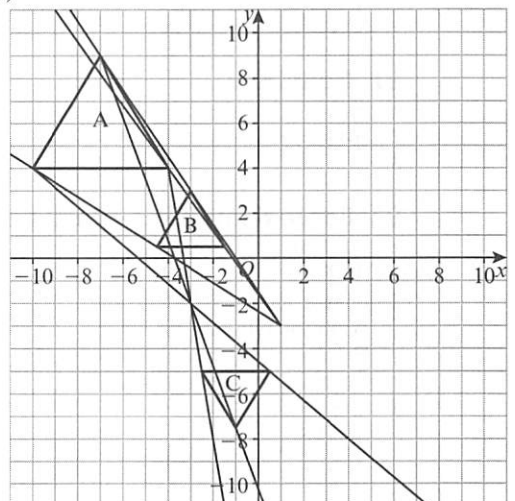   (c) Translation $\begin{bmatrix} 6 \\ -6 \end{bmatrix}$
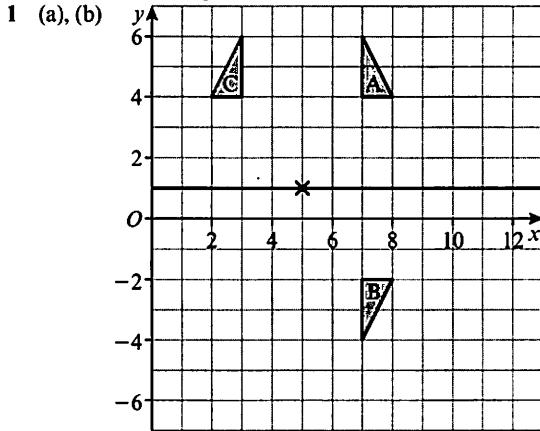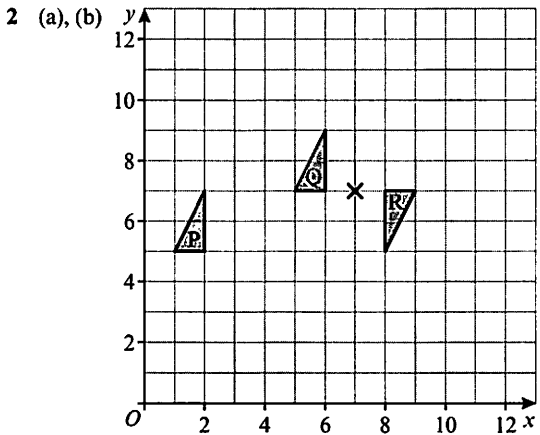
89. Enlargement

1. (a) 5
   (b)

2.

3. (a), (b)

90. Combining transformations

1 (a), (b)

(c) Reflection in line $x = 5$

2 (a), (b)

(c) Rotation 180° about (5, 6)

3 (a), (b)
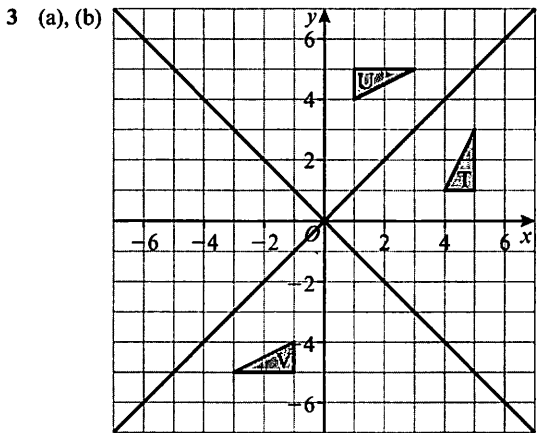
(c) Reflection in the line $y = -x$

91. Bearings

1 062°
2 304°
3 232°

92. Scale drawing and maps

1 (a) 15 m
   (b) 15 cm
2 Length: 800 cm ÷ 50 cm = 16
   Width: 400 cm ÷ 50 cm = 8
   Number of slabs needed 16 × 8 = 128
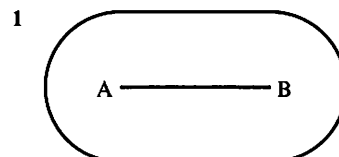   No, he does not buy enough slabs
3 (a) 53.5 km
   (b) 8 km

93. Constructions 1

1

2

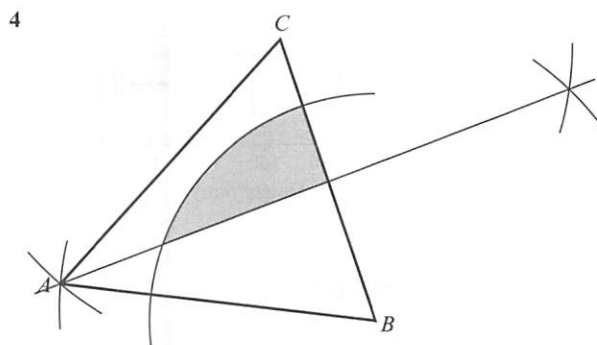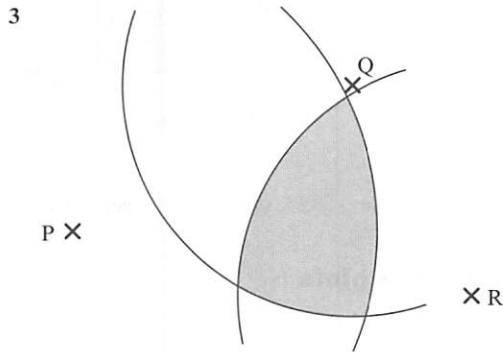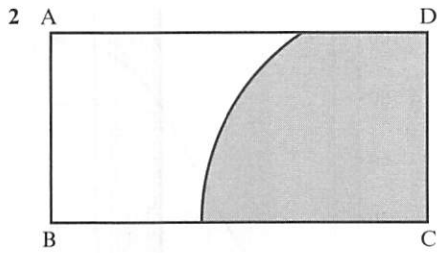3
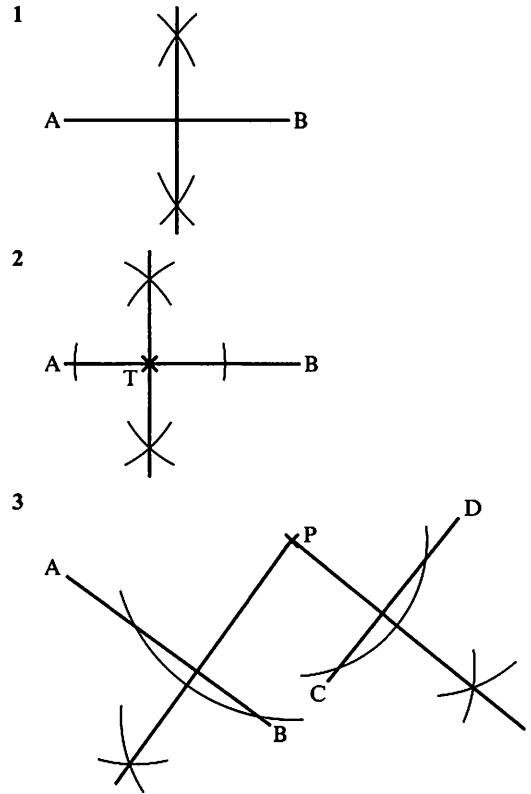
94. Constructions 2

1

2

3

4

95. Loci

1

146

2

3

4
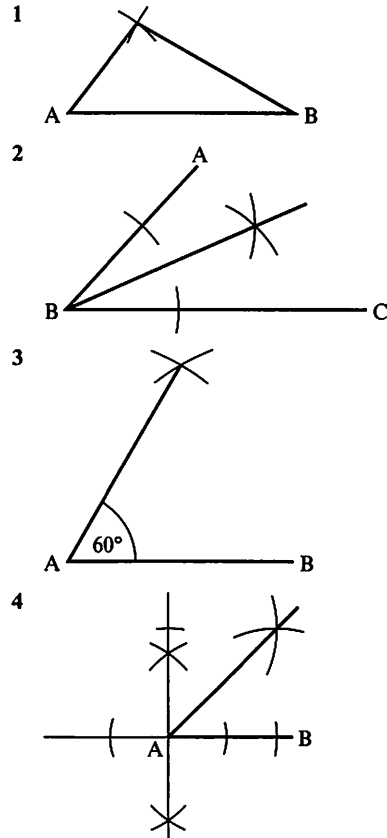
96. Congruent triangles

1 alternate parallelogram; SAS
2 BC is common; $BM = CL$; angle LBC = angle MCB; SAS
3 (a) $AB = AC$ (equilateral triangle); AD is common; angle ADC = angle ADB (90°); triangle ADC is congruent to triangle ADB; RHS
  (b) Since triangles ADB and ADC are congruent, $BD = DC$ so DC is half of BC. But, $BC = AC$ (equilateral triangle) so DC is half of AC.
4 Angle EDC = angle EBA (alternate); angle BAE = angle DCE (alternate); $DE = EB$; ABE and CDE are congruent; AAS

97. Similar shapes 1

1 (a) 25 (b) 18
2 35
3 12.5
4 5.6

98. Similar shapes 2

1 135
2 (a) 2430 (b) 18
3 5375
4 (a) 180 cm (b) 268 cm³

99. The sine rule

1 (a) 4.90 cm (b) 5.37 cm
2 (a) 29.3° (b) 34.3°
3 22.2 m
4 26.5 m

100. The cosine rule

1 (a) 9.60 cm (b) 9.85 cm
2 (a) 51.9° (b) 141.6°
3 16.0 cm
4 60.3 km

101. Triangles and segments

1 (a) 71.9 cm² (b) 72.1 cm²
2 (a) 6.23 cm² (b) 229 cm²
3 (a) 43.3 m² (b) 30.2 m²

102. Pythagoras in 3D

1 13 cm
2 13.3 cm
3 7.42 cm
4 Length of diagonal is 32.9 cm, so stick will not fit

103. Trigonometry in 3D

1 (a) 13 cm (b) 24.8°
2 58.0°
3 15.0 cm

104. Circle facts

1 80°
2 (a) (i) 90° (ii) Tangent to circle is 90°
  (b) 42°
3 Angle $OBA = 90 - x$
  Angle $OAB = 90 - x$
  Angle $AOB = 180° - (90 - x + 90 - x) = 2x$

105. Circle theorems

1 63° because angles in the same segment are equal
2 (a) 148° because the angle at the centre is twice the angle at the circumference
  (b) 106° because opposite angles in a cyclic quadrilateral add up to 180°
3 (a) 54° because tangent to circle is at 90°
  (b) 54° because angle in a semicircle is 90°
4 Angle ABC = 54° (alternate segment theorem)
  Angle BCA = 63° (base angles in an isosceles triangle are equal)
  Angle OCA = 36° (tangent to circle is at 90°)
  Angle OCB = 27°

106. Vectors

1 (a) $\binom{2}{5}$ (b) $\binom{-2}{-5}$ (c) $\binom{5}{3}$
  (d) $\binom{-5}{-3}$ (e) $\binom{5}{-6}$ (f) $\binom{-5}{6}$
2 (a) $\mathbf{a} + \mathbf{b}$ (b) $-\mathbf{a} - \mathbf{b}$
3 (a) $\mathbf{q} + \mathbf{p}$ (b) $-\mathbf{q} - \mathbf{p}$ (c) $\mathbf{p} - \mathbf{q}$ (d) $\mathbf{q} - \mathbf{p}$
4 (a) $\mathbf{a} + \mathbf{b}$ (b) $-\mathbf{a} - \mathbf{b}$

107. Vector proof

1 (a) (i) $2\mathbf{b}$ (ii) $2\mathbf{b} - \mathbf{a}$
  (b) $\overrightarrow{CA} = 2\mathbf{a} - 4\mathbf{b} = -2(2\mathbf{b} - \mathbf{a})$, this is a multiple of $(2\mathbf{b} - \mathbf{a})$ so CA is parallel to MN
2 (a) (i) $\mathbf{b} - \mathbf{a}$ (ii) $\frac{1}{5}(\mathbf{b} - \mathbf{a})$ (iii) $\frac{1}{5}(4\mathbf{a} - \mathbf{b})$
  (b) $\frac{1}{2}\mathbf{b} - \frac{4}{5}(\mathbf{b} - \mathbf{a}) = \frac{1}{10}(8\mathbf{a} - 3\mathbf{b})$
3 $\overrightarrow{AD} = \mathbf{c} - \mathbf{a}$
  $\overrightarrow{OE} = 2\mathbf{c} - 2\mathbf{a}$
  $\overrightarrow{OE} = 2(\mathbf{c} - \mathbf{a}) = 2\overrightarrow{AD}$
  Ratio of lengths $AD : OE$ is $1 : 2$

108. Problem-solving practice 1

1 $\frac{1}{3} \times \pi \times x^2 \times h = \frac{4}{3} \times \pi \times x^3$
  So $h = 4x$
2 (a) 37.5 km (b) 271°
3 $450 \div \left(\frac{3}{5}\right)^2 = 162$ cm²

109. Problem-solving practice 2

4 55.9°
5 $\overrightarrow{BC} = 4\mathbf{a} + 8\mathbf{b}$
  $\overrightarrow{DB} = 12\mathbf{a} - 8\mathbf{b}$
  $\overrightarrow{AE} = 3\mathbf{a} + 6\mathbf{b}$
  $\overrightarrow{BC}$ and $\overrightarrow{AE}$ are multiples of $\mathbf{a} + \mathbf{b}$, so lines BC and AE are parallel
6 29.6 cm²